Explorations in Feynman parameter space

Akshay Yelleshpur

Princeton University

June 12, 2018

Amplitudes 2018 summer school
UC Davis

Based on

1712.0991 by Nima Arkani-Hamed and Ellis Ye Yuan
and work in progress with Nima Arkani-Hamed

Introduction

Momentum space

$$I_5 = \int \frac{d^4x}{(x-x_1)^2(x-x_2)^2(x-x_3)^2(x-x_4)^2(x-x_5)^2}$$
$$= \frac{r^\epsilon}{s_{23}s_{34}s_{45}}\left[\frac{1}{\epsilon^2} + 2Li_2\left(1-\frac{s_{23}}{s_{51}}\right) + 2Li_2\left(1-\frac{s_{45}}{s_{12}}\right) - \frac{\pi^2}{6}\right] + \text{cyclic} + \mathcal{O}(\epsilon)$$

Branch point at $s_{23} = (x_1 - x_3)^2 = 0$. *Log* discontinuity of Li_2.

Feynman parameter space

$$I_5 = \int \langle \alpha d^4\alpha \rangle \frac{\sum_i \alpha_i}{(\sum_{i<j} \alpha_i\alpha_j(x_i-x_j)^2)^3}$$
$$= \int \langle \alpha d^4\alpha \rangle \frac{L.\alpha}{(\alpha Q\alpha)^3}$$
$$L = (1,1,1,1,1) \qquad \alpha = (\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5) \qquad Q_{ij} = (x_i - x_j)^2$$

How do we calculate the discontinuity? How do we locate the branch points?

Spherical Residues

"Spherical residue $\rightarrow$ prescription to calculate discontinuities.

Bring denominator to a "canonical form". Eliminate terms linear in α_1 and α_3.

$$\alpha_1 = \frac{\hat{\alpha}_1}{\sqrt{Q_{13}}} - \frac{Q_{35}\alpha_5}{Q_{13}} \qquad \alpha_3 = \frac{\hat{\alpha}_3}{\sqrt{Q_{13}}} - \frac{Q_{14}\alpha_4}{Q_{13}}$$

$$\int \langle \alpha d^4\alpha \rangle \frac{L.\alpha}{(\alpha.Q.\alpha)^3} \rightarrow \int \langle \hat{\alpha} d^4 \hat{\alpha} \rangle \frac{\hat{L}.\hat{\alpha}}{(\hat{\alpha}_1\hat{\alpha}_3 + f(\alpha_k))^3} \quad (k \neq 1,\, 3)$$

Integrate over the contour $\hat{\alpha}_1 = \hat{\alpha}_3^*$ $\rightarrow$ Spherical contour.

Result of integration = "Residue". Branch points are $Q_{13} = 0$.

Cuts 2 propagators $(x - x_1)^2 = (x - x_3)^2 = 0$.

Double spherical residues $(\alpha_1, \alpha_3, \alpha_2, \alpha_4) \rightarrow$ Cuts four propagators

IR Divergences

Scalar pentagon is not finite. IR divergences ($\frac{1}{\epsilon^2}$ terms).

Characterization in Feynman parameter space?

$$\alpha_2 = \sqrt{\alpha_3}\,\rho\, e^{\tau} \qquad\qquad \alpha_4 = \sqrt{\alpha_3}\,\rho\, e^{-\tau}$$

Dominant term for large $\alpha_3 \to$ coefficient of $\frac{1}{\epsilon^2}$.

$$\frac{1}{2}\int d\alpha_3\, d\rho^2\, d\alpha_1 d\tau \frac{\alpha_3^2}{\alpha_3^3(\rho^2(x_2 - x_4)^2 + \alpha_1(x_1 - x_3)^2 + (x_1 - x_5)^2)^3}$$

$$= \int dlog(\alpha_3)\, dlog(\frac{\alpha_2}{\alpha_4})\frac{1}{s_{23}s_{34}s_{45}}$$

Residue Theorems

Double Spherical Residue = Leading Singularity.

$\implies$ Must obey the Global Residue Theorem (GRT) $\rightarrow$ Sum of all global residues is zero.

Naively,

$$\sum_{i,j}(a\, b\, i\, j) = 0 \qquad\qquad a,\ b \in \{1,2,3,4,5\}$$

$$(2435) + (2413) = \frac{1}{Q_{24}\,Q_{35}\,Q_{31}} = \frac{1}{s_{23}s_{34}s_{45}} \neq 0$$

Missing residues?

Collinear regions require $(x - x_2)^2 = (x - x_3)^2 = (x - x_4)^2 = 0$.

Only three poles but completely localized x.

IR Scaling $\rightarrow$ composite residue.

THANK YOU FOR YOUR ATTENTION