Scattering Forms, Worldsheet Forms and Amplitudes from Subspaces

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Based on:

arXiv: 1711.09102 Nima Arkani-Hamed, Yuntao Bai, Song He, Gongwang Yan

arXiv: 1803.11302 Song He, Gongwang Yan, Chi Zhang, Yong Zhang

Story of associahedron

- World sheet configuration region:

$$M_{0,n}^{+}:\{[0=\sigma_1<\cdots<\sigma_{n-1}=1<\sigma_n=+\infty]_{SL(2,C)}\}$$

- Kinematic associahedron:

$$A_n := \{s_{(planar)}\geq 0\}\cap H_n,$$

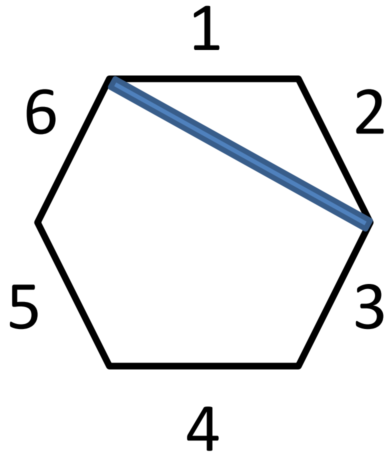
$$H_n=\{s_{ij}=-c_{ij}<0, 1\leq i<j-1\leq n-2\}$$

- "pushforward" :

$$\Omega(M_{0,n}^{+})=PT(12...n)\frac{d\sigma_1\wedge..\wedge d\sigma_n}{vol(SL(2,C))}\xrightarrow[\text{of scattering eqns}]{\text{sum over solutions}}\Omega_{12...n}^{\Phi^3}$$

- Pullback: $\Omega_{12...n}^{\Phi^3}|_{H_n}=m_{\Phi^3}\left(12..n|12..n\right)vol(H_n)$

Kinematic space $K_n=\{s_{ij}\}$ spanned by "planar" variables:



$$s_{12}=s_{3456}$$

$$s_{14}=s_{1234}-s_{123}-s_{234}+s_{23}$$

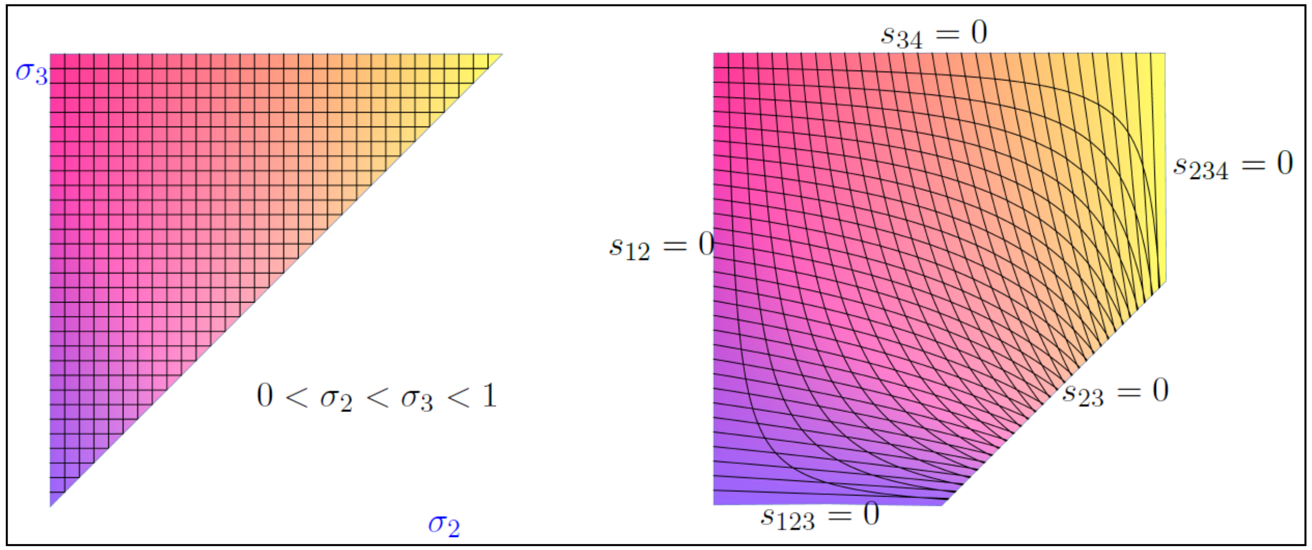
Scattering Equation as a map

- Rewriting the equations $E_a := \sum_{b \neq a} \frac{s_{ab}}{\sigma_{ab}}$ according to H_n : $\left(\begin{array}{l} \sigma_{ab} := \sigma_a - \sigma_b \\ \sigma_n \to \infty \end{array} \right)$

$$\sum_{a=1}^{i} E_a = \frac{s_{i(i+1)}}{\sigma_{i(i+1)}} + \sum_{\substack{1 \le a \le i < b \le n-1 \\ (a,b) \neq (i,i+1)}} \frac{s_{ab}}{\sigma_{ab}}$$

- Which interpret the eqns as a map $M_{0,n}^{+} \mapsto A_n$:

$s_{i(i+1)} = \sigma_{i(i+1)} \sum_{\substack{1 \le a \le i < b \le n-1 \\ (a,b) \neq (i,i+1)}} \frac{c_{ab}}{\sigma_{ab}}$, validating the "pushforward"



- Furthermore, rewriting shows:

$$J_{H_n} := \det'\left(\frac{\partial E}{\partial s}\Big|_{H_n}\right)$$

$$= PT(12...n) = \text{"}\Omega(M_{0,n}^{+}) / \frac{d\sigma_1 \wedge .. \wedge d\sigma_n}{vol(SL(2,C))}\text{"}$$

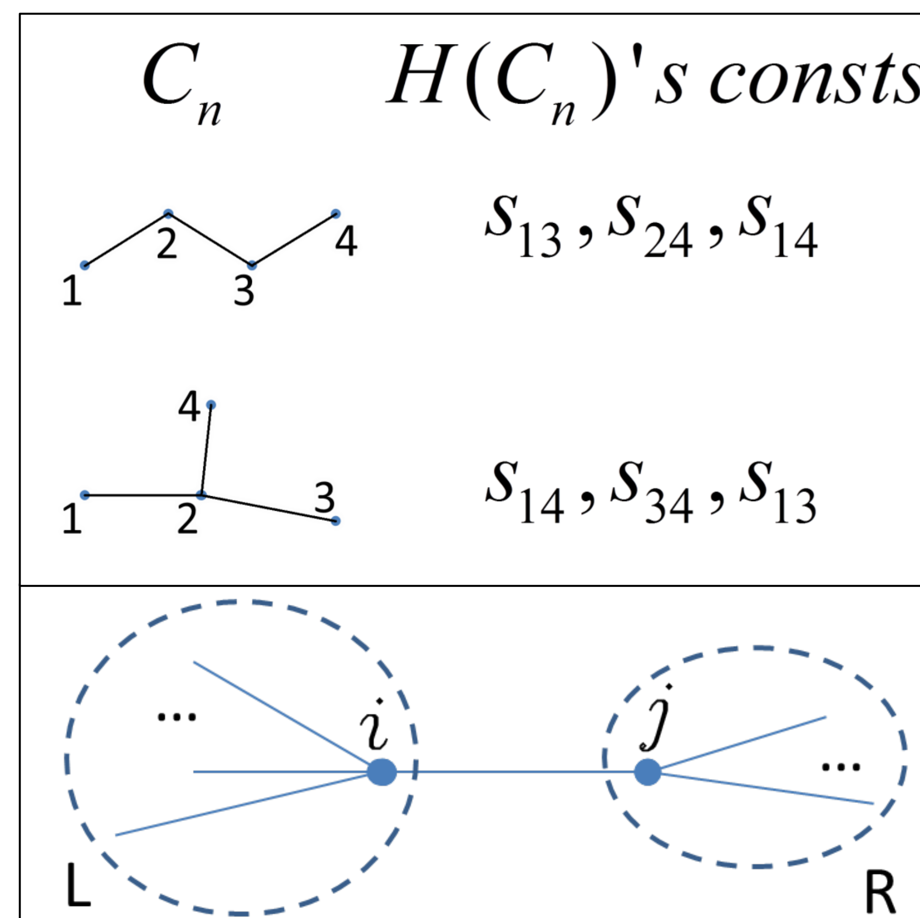
Generalization: subsapces $H(C_n)$

- Fix $(n-2)(n-3)/2$ of $\{s_{ij} \mid 1 \le i < j \le n-1\}$ to be negative constants such that a graph C_n with edge ij corresponding to unfixed s_{ij} is a Cayley (tree) graph with $(n-1)$ vertices.
- Rewriting: $(\sigma_n \to \infty)$

$$\sum_{a \in L} E_a = \frac{s_{ij}}{\sigma_{ij}} + \sum_{\substack{a \in L, b \in R \\ (a,b) \neq (i,j)}} \frac{s_{ab}}{\sigma_{ab}}$$

- World sheet form:

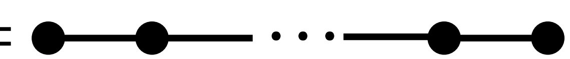
$$\omega_{H(C_n)} := \det'\left(\frac{\partial E}{\partial s}\Big|_{H(C_n)}\right) d^{n-3}\sigma \xrightarrow{(\sigma_1=0,\sigma_{n-1}=1)} \pm \prod_{ij \in edge(C_n)} \sigma_{ij}^{-1} \wedge d\sigma_2 \wedge \cdots \wedge d\sigma_{n-2}$$



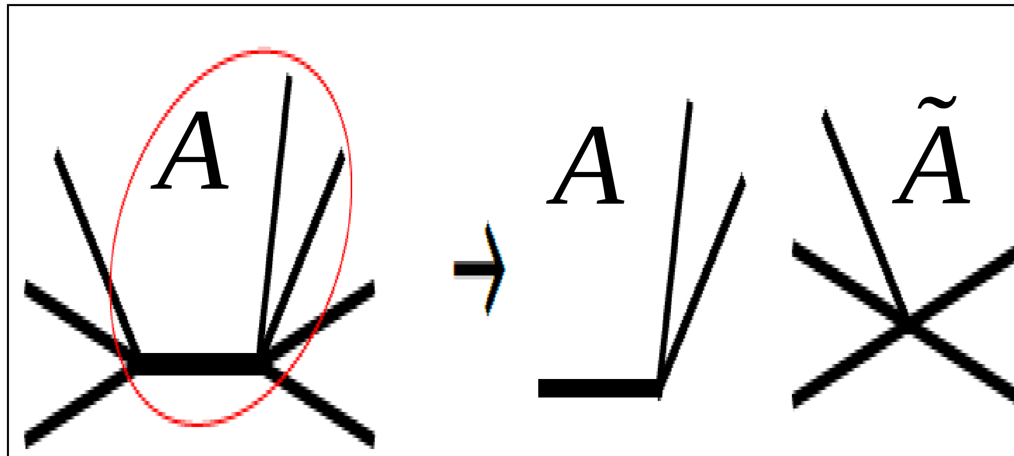
Cayley polytopes: $P(C_n)$

- $P(C_n) := \Delta(C_n) \cap H(C_n)$, where
  $\Delta(C_n) := \{s_A \geq 0 \mid A \subset C_n \ subtree\}$

Associahedron:

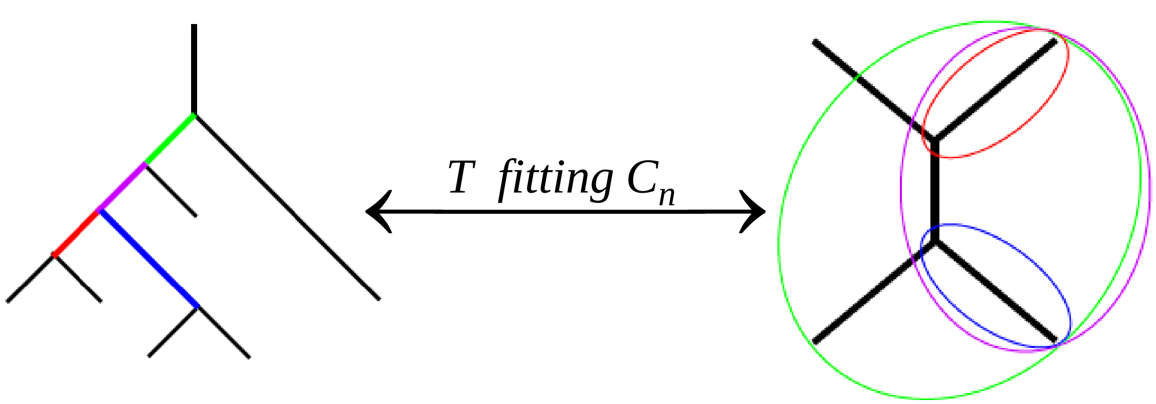
$C_n =$ 

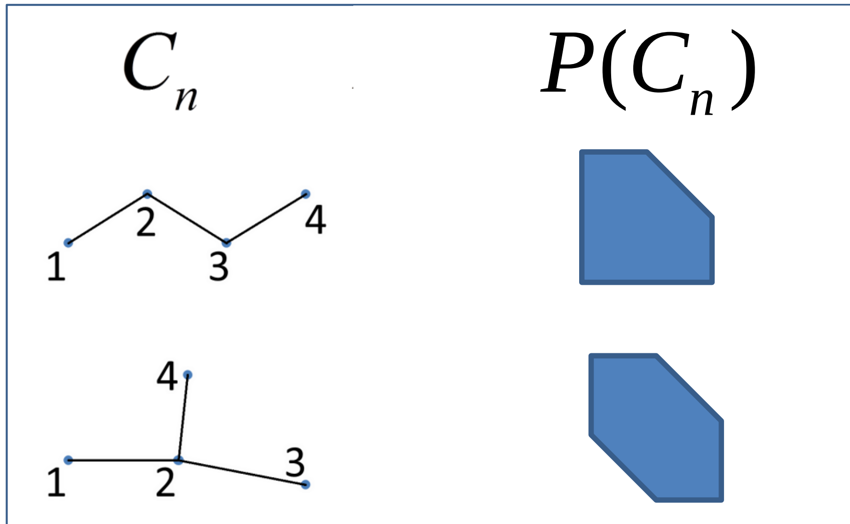
- Factorizable boundaries
  $$P(C_n)|_{s_A=0} = P(A) \times P(\tilde{A})$$



- The canonical form

$$\Omega(P(C_n)) = \pm\Big(\sum_{T \ fitting \ C_n} \prod_{T's \ propagator} s^{-1}\Big) vol(H(C_n))$$



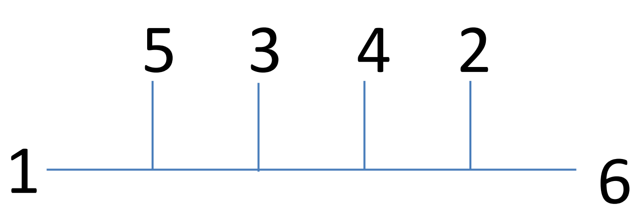


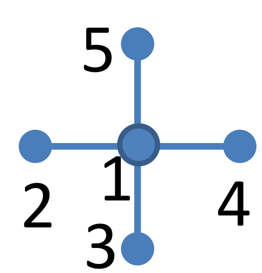
“Pushforward” and pullback

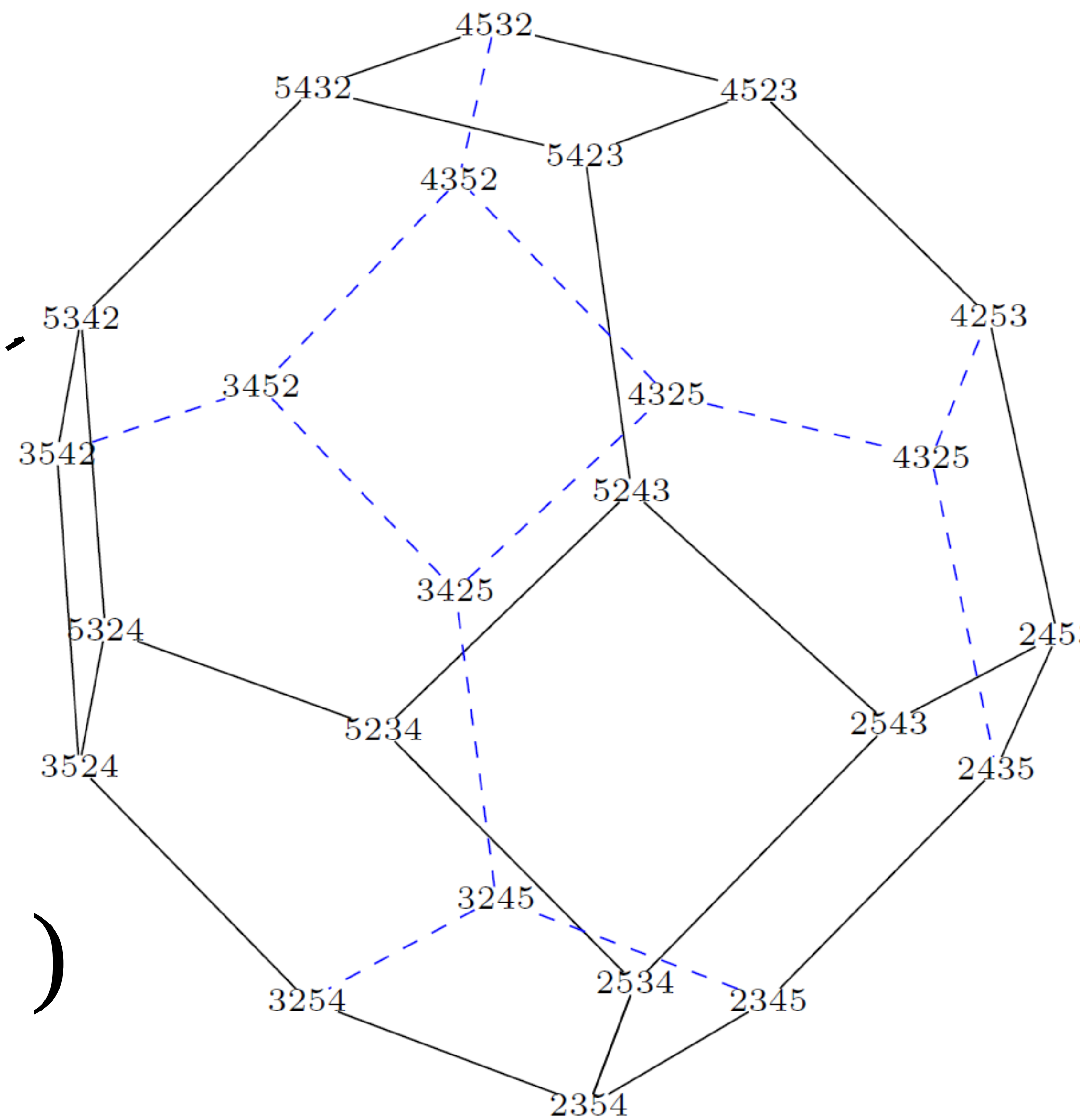
- $$\omega_{H(C_n)} \xrightarrow[\text{of scattering eqns}]{\text{sum over solutions}} \Omega_{H(C_n)} = \sum_{T \text{ fitting } C_n} \pm \bigwedge_{T\text{'s propagator}} d\log s$$
- $$\Omega_{H(C_n)}\big|_{H(C_n)} = \Omega(P(C_n)) = \pm\Big(\sum_{T \text{ fitting } C_n} \prod_{T\text{'s propagator}} s^{-1}\Big) vol(H(C_n))$$

Each term corresponding to a vertex of $P(C_n)$

“5342” means this Feynman graph:



$P($  $)$



Thank you for attention!