Unifying Tree Super-Amplitudes in 6D: Branes, SYM, and SUGRA

Matthew Heydeman\textsuperscript{1}, Alfredo Guevara\textsuperscript{2,3}, Sebastian Mizera\textsuperscript{2}

\textsuperscript{1}California Institute of Technology
\textsuperscript{2}Perimeter Institute for Theoretical Physics
\textsuperscript{3}CECs Valdivia

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Main ideas

- Why 6D? The existence of chiral gauge theories in 6D explains and unifies many properties of 4D supersymmetric gauge theories.
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- Goal: Write down the complete tree-level S-matrix for maximally supersymmetric gauge theories, gravity, and effective field theory in six spacetime dimensions.
Main ideas

- Why 6D? The existence of chiral gauge theories in 6D explains and unifies many properties of 4D supersymmetric gauge theories.

- Goal: Write down the complete tree-level S-matrix for maximally supersymmetric gauge theories, gravity, and effective field theory in six spacetime dimensions.

- The main tool is to boost Witten’s twistor string from $4 \rightarrow 6$ dimensions. The $n$-particle amplitude is an integral over the $n$-punctured Riemann sphere (possibly with other moduli $\mathcal{M}$):

\[
A_n \sim \int \frac{d^n\sigma d\mathcal{M}}{\text{Vol}(G)} \langle \text{String Correlation Function} \rangle
\]

We find a unified description of 6D theories in this form.
Open Strings and Dp-branes

- Quantization of open strings on Dirichlet $p$-brane $\rightarrow p + 1$ dimensional vector multiplet. Maximal SUSY for spins $\leq 1$. 

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Quantization of open strings on Dirichlet $p$-brane $\rightarrow p + 1$ dimensional vector multiplet. Maximal SUSY for spins $\leq 1$.

For $\ell_s \rightarrow 0$, this is a free theory, but finite $\ell_s$ corrections give supersymmetric Dirac-Born-Infeld theory (brane action):

$$S \sim T \int d^{p+1}x \sqrt{-\det(g + F)}$$
Multiple Branes $\rightarrow$ Super Yang-Mills

$N$ Dp Branes

KLT $\rightarrow$ $p + 1$-dimensional Maximal Supergravity
(Ex: $\mathcal{N} = 4$ SYM $\rightarrow$ $\mathcal{N} = 8$ SUGRA)

Scalar Vevs
Twistor strings and rational maps

- Witten’s observation: Scattering of $\mathcal{N} = 4$ SYM (field theory) computed exactly by a topological string theory! Open B-Model on supertwistor space.
- Amplitude supported on punctured D1 strings wrapping curves, integrate correlator over punctures and moduli of maps:

$$
\lambda^\alpha(z) = \sum_{k=0}^{d} \rho^\alpha_k z^k \\
\tilde{\lambda}^{\dot{\alpha}}(z) = \sum_{k=0}^{\tilde{d}} \tilde{\rho}^{\dot{\alpha}}_k z^k
$$

$$
d + \tilde{d} = n - 2 \\
\tilde{d} - d = \text{helicity violation}
$$
A Six-Dimensional Surprise

- In classifying super-Poincare and superconformal algebras, one finds there are actually *two different* maximal theories of spin-1 fields!
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- In classifying super-Poincare and superconformal algebras, one finds there are actually \textit{two different} maximal theories of spin-1 fields!

- Chiral and anti-chiral pseudo-real spinors: \((p, q)\) SUSY. 
  
  \((1, 1) \rightarrow \text{D5-brane and 6D SYM. But } (2, 0) \rightarrow \text{self-dual (chiral) 2-form gauge field } B_{\mu\nu}. \ H = dB = \ast H \rightarrow \text{no candidate action!} \)
The challenge:

- The Witten twistor string relied crucially on the 4D spinor helicity variables to construct maps. In 6D there are no helicity sectors due to the little group.
- The simplest 6D generalization turn out to be the single D5 and M5-brane EFTs. Maximal Yang-Mills and SUGRA are harder due to the structure of the maps.
- Apply to lower dimensions: 5D SYM/SUGRA and 4D Coulomb Branch amplitudes
Towards Rational Maps: Spinor Variables in 6D

- Momentum vectors can be described as bispinors of \( \text{Spin}(5, 1) \sim SU^*(4) \), \( p^\mu \sim p^{AB} \) with \( A, B = 1, \ldots, 4 \).

- Little group = \( SU(2) \times SU(2) \). We introduce \( \lambda^A_{ia} \) such that

\[
p^{AB}_i = \langle \lambda^A_i \lambda^B_i \rangle = \epsilon_{ab} \lambda^{A,a}_i \lambda^{B,b}_i = \lambda^A_i \lambda^-_i - \lambda^-_i \lambda^+_i.
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$$p_{i}^{AB} = \langle \lambda_{i}^{A} \lambda_{i}^{B} \rangle = \epsilon_{ab} \lambda_{i}^{A,a} \lambda_{i}^{B,b} = \lambda_{i}^{A+} \lambda_{i}^{B-} - \lambda_{i}^{A-} \lambda_{i}^{B+}.$$
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- Lorentz invariants:

$$\langle \lambda^a_i \lambda^b_j \lambda^c_k \lambda^d_l \rangle = \epsilon_{ABCD} \lambda^A_i a \lambda^B_j b \lambda^C_k c \lambda^D_l d.$$
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- E.g., 4 gluon scattering:
  \[ \mathcal{A}(g_1^{a\hat{a}}, g_2^{b\hat{b}}, g_3^{c\hat{c}}, g_4^{d\hat{d}}) = \frac{\langle 1^a 2^b 3^c 4^d \rangle [1^{\hat{a}} 2^{\hat{b}} 3^{\hat{c}} 4^{\hat{d}}]}{s_{12} s_{23}}. \]
Promote spinor variables to polynomials. Construct the null map:

$$z \in \mathbb{CP}^1 \longrightarrow p^{AB}(z) = \langle \rho^A(z), \rho^B(z) \rangle$$
The most natural choice consistent with $SL(2, \mathbb{C})$ is

\[
\rho^{A,a}(z) = \sum_{k=0}^{d=\left\lceil \frac{n}{2} \right\rceil - 1} \rho_{k}^{A,a} z^{k}.
\]

where for odd $n$ we require the degenerate condition $\rho_{d}^{A,a} = \omega^{A} \xi^{a}!$

These maps are to be determined by the condition

\[
p_{i}^{AB} = \frac{\langle \rho^{A}(\sigma_i), \rho^{B}(\sigma_i) \rangle}{\prod_{j \neq i} \sigma_{ij}}
\]

which also fixes the punctures $\{\sigma_i\}$ in $\mathbb{CP}^1$ (i.e. Scattering Equations)
6D Amplitude

We are now ready to construct the amplitudes for our favorite 6D theories by integrating over the moduli space of maps!

\[ A_{6D} = \int \frac{\prod d\sigma_i d\rho_k}{\text{Vol}(G)(\prod \sigma_{ij})^2} \prod_{i=1}^{n} \delta^6 \left( p_{i}^{AB} - \frac{\langle \rho^A(\sigma_i), \rho^B(\sigma_i) \rangle}{\prod_{j \neq i} \sigma_{ij}} \right) \times \mathcal{I_L}\mathcal{I_R} \]

where \( \text{Vol}(G) \) stands for the redundancies of the moduli space. For odd \( n \) an enlarged symmetry group emerges. We find that the delta functions completely localize the integration variables on \((n - 3)!\) points of the moduli.
M5, D5-Branes and SYM

The integrands $\mathcal{I}_{L,R}$ depend on the theory. They carry the fermionic components of the amplitude. This are defined in analogous way to the bosonic delta functions:

$$\Delta_{F} = \int \prod_{k=0}^{d} d\chi_{k} \prod_{i=1}^{n} \delta^{4} \left( q_{i}^{A} - \frac{\langle \rho^{A}(\sigma_{i}), \chi(\sigma_{i}) \rangle}{\prod_{j \neq i} \sigma_{ij}} \right)$$

where $q_{i}^{A}$ is now the supermomenta of the $i$-th particle.

We have half-integrands:

$$\mathcal{I}^{N=\langle 2,0 \rangle} = \text{Pf} \cdot A_{n} \times \Delta_{F}^{2}, \quad \mathcal{I}^{N=\langle 1,1 \rangle} = \text{Pf} \cdot A_{n} \times \Delta_{F} \tilde{\Delta}_{F}$$

$$\mathcal{I}^{\text{abelian}} = (\text{Pf} \cdot A_{n})^{2}, \quad \mathcal{I}^{\text{non-abelian}} = \frac{\text{Tr} \left( T^{a_{1}} T^{a_{2}} \cdots T^{a_{n}} \right)}{\sigma_{12} \sigma_{23} \cdots \sigma_{n1}}.$$

where $\text{Pf} \cdot A_{n}$ is constructed from minors of $(A_{n})_{ij} := \frac{p_{i} \cdot p_{j}}{\sigma_{ij}}$.
Use the half-integrands as building blocks for amplitudes:

\[
\int d\mu_{\text{maps}} \mathcal{I}_L \mathcal{I}_R \quad \mathcal{I}_L \quad \mathcal{I}_R
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N = (2,2) SUGRA is a double-copy of N = (1,1) SYM.

Perturbative amplitudes in non-abelian N = (2,0) theory should vanish; our formula computes some other non-abelian object with on-shell supersymmetry.
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- Perturbative amplitudes in non-abelian \mathcal{N} = (2, 0) theory should vanish; our formula computes some other non-abelian object with \mathcal{N} = (2, 0) on-shell supersymmetry
We find new amplitudes for mixed theories using the half-integrand:

\[ I^{\text{semi-abelian}} = \frac{\text{Tr}(T^{a_1}T^{a_2}\cdots T^{a_k})}{\sigma_1\sigma_2\cdots\sigma_k}(\text{Pf}A_{k+1,...,n})^2, \]
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$$\mathcal{A}^{\text{D5-branes } \oplus \text{SYM}} = \int d\mu_{\text{maps}} \mathcal{I}^{\text{semi-abelian}} \mathcal{I}^\mathcal{N}=(1,1)$$
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\[ \mathcal{A}_{D5-\text{branes} \oplus SYM} = \int d\mu_{\text{maps}} I_{\text{semi-abelian}} I^N=(1,1) \]

gives an S-matrix of an interacting theory between the abelian and non-abelian sectors of D5-branes.
Even if you couldn't care less about 6D, we have something for you: Embed 4D massive momenta into 6D massless ones as follows

\[ \lambda^{A,a} = \begin{pmatrix} \frac{m_{\mu\alpha}}{\langle \mu \lambda \rangle} & \lambda_\alpha \\ \tilde{\lambda} \tilde{\alpha} & \frac{m_{\tilde{\mu} \tilde{\alpha}}}{[\tilde{\lambda} \tilde{\mu}]} \end{pmatrix} \implies p_{\alpha \dot{\alpha}} = \lambda_\alpha \tilde{\lambda} \dot{\alpha} + m^2 \frac{\mu_{\alpha \tilde{\mu} \tilde{\alpha}}}{\langle \lambda \mu \rangle [\tilde{\lambda} \tilde{\mu}]} . \]
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- We can do concrete calculations, for instance, 4-pt amplitude of W-bosons and gluons:

\[
A(W_1^+, \overline{W}_2^-, g_3^-, g_4^-) = \frac{m^2 [1\mu]^2 \langle 34 \rangle^2}{[2\mu]^2 s_{12} (s_{23} - m^2)}.
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- Leaves many applications for computing loop integrands
If you want to know more check out:

- “M5-Brane and D-Brane Scattering Amplitudes”
  MTH, J.H. Schwarz, C. Wen [hep-th/1710.02170]

- “The S Matrix of 6D Super Yang–Mills and Maximal Supergravity from Rational Maps”
  F. Cachazo, AG, MTH, SM, J.H. Schwarz, C. Wen [hep-th/1805.11111]

- Alfredo’s and Matt’s poster at SLAC next week
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