Benchmarking planar five-parton two-loop QCD amplitudes with numerical unitarity

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Motivation

Precision era at the LHC

- No direct detection of new physics $\implies$ zoom in into data
- High order calculations, i.e. NNLO are required to achieve $\approx 1\%$ level accuracy
  theory predictions for signal and background

State of the art

- Most of $2 \rightarrow 2$ processes are available at NNLO, but many interesting processes have $> 2$ particles in the final state
- Handling IR divergences for $> 2$ particles is very challenging, active research by many groups
- Huge effort towards computation of multi-scale Feynman integrals
- $2 \rightarrow 3$ two-loop amplitude frontier is being actively attacked and first simplest amplitudes have been benchmarked

We focus on integrand reduction of two-loop amplitudes with numerical unitarity method.
The Standard Approach to General Two-loop Amplitudes

- **Feynman diagrams**
- **Tensor reduction** [Passarino, Veltman '79]
- **IBPs** [Tkachov, Chetyrkin '81]

**Sum of master integrals**

\[
A = \sum_{\Gamma \in \Delta} \sum_{i \in M_{\Gamma}} c_{\Gamma,i}(D, D_s) I_{\Gamma,i}
\]

- **Differential equations** [Kotikov '91; Remiddi '97; Gehrmann, Remiddi '01; Henn '13]
- **Integrated amplitude**

**Challenges:**

- Large intermediate expressions
- Generating IBP relations is **practically difficult**

**Two-loop numerical unitarity** tries to avoid these issues:

- Only a **restricted set of IBP relations** is required for each topology
- **Implicit numerical reduction** to master integrals
- **Full numerical framework** avoids expression bloat
1. Take an ansatz for loop-amplitude integrand, decomposing into master \((M_\Gamma)\) and surface \((S_\Gamma)\) integrands [ita ’15].

\[
A(\ell_l) = \sum_{\text{Topologies } \Gamma} \sum_{i \in M_\Gamma \cup S_\Gamma} \frac{c_{\Gamma,i} m_{\Gamma,i}(\ell_l)}{\prod_{\text{props } j} \rho_j}.
\]

2. For each topology build linear systems (cut equations) for master/surface coefficients \(c_{\Gamma,i}\) by putting loop momenta on-shell.

\[
\sum_{i \in M_\Gamma \cup S_\Gamma} c_{\Gamma,i}(D, D_s) m_{\Gamma,i}(\ell_\Gamma^\Gamma) = - \sum_{\text{ancestors } \Gamma'} \frac{N(\Gamma', \ell_l^\Gamma)}{\prod_{k \in P_{\Gamma'} \setminus P_{\Gamma}} \rho_k(\ell_l^\Gamma)}
\]

3. Invert linear systems (e.g. by PLU or QR factorization) for given kinematics, \(D\) and \(D_s\)

4. Reconstruct rational functions of \(D\) and \(D_s\) by sampling \(\Rightarrow\) master coefficients directly from on-shell data.

5. Combine with master integrals \(\Rightarrow\) integrated amplitude
Status

The BH2 Project
We are constructing a C++ framework for $D$-dimensional multi-loop numerical unitarity. We implement algorithms suitable for multi-precision floating point as well as exact arithmetics (finite fields $\rightarrow$ rational numbers).

Collaboration
Samuel Abreu, Jerry Dormans, Fernando Febres-Cordero, Harald Ita, Matthieu Jaquier, Ben Page, Evgenij Pascual, VS

Results so far

- 4 point Yang-Mills amplitudes [arXiv:1703.05273]: reproduced analytic results from literature [Bern, De Freitas, Dixon '02]
- benchmark 5 point Yang-Mills amplitudes [arXiv:1712.03946] (see also [Badger et al., arXiv:1712.02229])
- Reproduced known $N_f$-contributions to 4-gluon amplitudes
Outlook

What’s next?

- Extension to **full QCD spectrum** and beyond. Challenges:
  - dim. reg with fermions in numerical framework [arXiv:1803.11127]
  - no square roots (of scalar products) allowed for exact arithmetics (as in \( \ell[D] \))
  - efficient colour decomposition with quarks

- Functional reconstruction of full kinematical dependence of integral coefficients

- **Numerical stability** and **performance** improvements ⇒ *integrated* virtual matrix elements

- **Non-planar topologies**: (multiple) non-coloured particles in the final state; sub-leading colour contributions

- **Long term goal**: combine with other bits of NNLO computation to deliver full NNLO precise predictions for multi-scale processes

Stay tuned!
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