Conformal Symmetry and Feynman Integrals

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Conformal symmetry

- The consequences of conformal symmetry in position space are well studied
- Our goal: application to scattering amplitudes
  - work in momentum space
  - on-shell massless configuration $p_i^2 = 0$
- Generator of conformal boosts becomes a 2nd order operator

$$K_\mu = \sum_{i=1}^{n} \left[-p_{i\mu} \Box_{p_i} + 2 p_i^\nu \frac{\partial}{\partial p_i^\nu} \frac{\partial}{\partial p_i^\mu} + 2(D - \Delta_i) \frac{\partial}{\partial p_i^\mu} \right]$$

- We cannot Fourier-transform from position to momentum space

Collinear anomaly

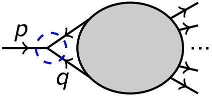
- Consider naïvely conformal Feynman integrals:
  conformal symmetry may be broken in massless configurations

  [Chicherin, Sokatchev 2017]

- E.g. $D = 6$ scalar Φ^3 theory
  - Contact anomaly

$$K_\mu \frac{1}{q^2(q+p)^2} = 4i\pi^3 p_\mu \int_0^1 d\xi \xi(1-\xi)\delta^{(6)}(q+\xi p)$$

on-shell corner
$p^2 = 0$



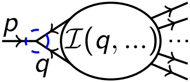
  - Localized on the collinear configuration

$$q = -\xi p, \qquad \xi \in [0,1]$$

Powerful anomalous conformal Ward identities

- The contact anomaly localizes a loop-integration

$$K_\mu \int d^6 q$$



$$\propto p_\mu \int_0^1 d\xi \xi (1-\xi) \mathcal{I}(q = -\xi p, \ldots)$$

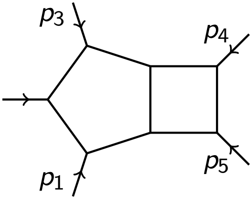
- System of linear non-homogeneous 2$^{\text{nd}}$-order DEs

$$K_\mu \delta^{(6)}(P) \mathcal{I}^{(\ell)} = \delta^{(6)}(P) \mathcal{A}_\mu^{(\ell-1)}$$

  - $\mathcal{I}^{(\ell)}$ ℓ-loop Feynman integral
  - $\mathcal{A}_\mu^{(\ell-1)}$ anomaly, 1-fold integration of $\ell - 1$-loop integrals

- We follow the bootstrap approach:
  1. write an ansatz for the (**symbol** of the) $\mathcal{I}$
  2. fix the coefficients through Ward identities

A first application: 6D penta-box

$\mathcal{I}_5^{(2)} =$ 

finite
planar
even under complex conjugation
graph symmetry $\{1 \leftrightarrow 3, 4 \leftrightarrow 5\}$

- Planar pentagon alphabet $\mathbb{A}_\mathrm{P} = \{\alpha_i\}_{i=1,\ldots,26}$

  [Gehrmann, Henn, Lo Presti 2015]

- Ansatz for the symbol is

$$\mathcal{S}\left[\mathcal{I}_5^{(2)}\right] = \frac{1}{\sqrt{\Delta}} \sum_{I=(i_1,\ldots,i_5)} c_I \left(\alpha_{i_1} \otimes \ldots \otimes \alpha_{i_5}\right), \quad \alpha_i \in \mathbb{A}_\mathrm{P}$$

where $\Delta = \det\left(2p_i \cdot p_j\right)$ is the Gram determinant

▷ **All coefficients are fixed**!
▷ Only **one projection** of the Ward identities is needed

Outlook

- Upgrade bootstrap to function level (Goncharov polylogarithms) $\Rightarrow$ fix "beyond the symbol" terms
- Extension to super-conformal symmetry

  [Chicherin, Henn, Sokatchev 2018]

  - Wess-Zumino model of $\mathcal{N} = 1$ massless supersymmetric matter
  - 1^{st} order Ward identities $\Rightarrow$ even more powerful!