Massless scattering amplitudes in 2d field theory

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based on work in progress with Arkady Tseytlin and Ben Hoare
Motivation

- Want to... classify integrability of sigma models on (1+1)d Minkowski space

\[ L = \partial_\mu X^i \partial_\nu X^j \eta^{\mu\nu} G_{ij}(X) + \partial_\mu X^i \partial_\nu X^j \epsilon^{\mu\nu} B_{ij}(X) \]

- Integrability = Lax connection \(\implies\) solvable EOM

- Very difficult!
Factorized scattering

**Theorem:**
The S-matrix of a relativistic **massive** integrable model has:
- No particle production
- Factorized scattering

What about massless models?! S-matrix **not well defined**

Want to design formal ‘S-matrix-type’ object that satisfies the theorem. (Tree-level only!)

Recent interest:
- [Dubovsky, Flauger, Gorbenko, 12]
- [Cooper, Dubovsky, Gorbenko, Mohsen, Storace, 15]
- [Wulff, 18]
- [Gabai, Mazac, Shieber, Vieira, 18]
Integrable Examples:

**$S^n$ sigma model**

$$\mathcal{L} = \frac{\partial y^i \partial y^i}{(1 + \frac{\lambda^2}{4} y^j y^j)^2} \quad (i, j = 1, \ldots, n)$$

Symmetric coset space: $S^n = SO(n+1)/SO(n) \rightarrow$ integrable

**Nappi model**

$$\mathcal{L} = (\partial A)^2 + (\partial B)^2 + (\partial C)^2 + g \, \epsilon^{\mu\nu} A \, \partial_\mu B \, \partial_\nu C$$

Classically equivalent to $S^3 \cong SU(2)$ sigma model!

Lax connection constructed $\rightarrow$ integrable

[Zakharov, Mikhailov, 78]

It was claimed to have particle production

[Nappi, 80]

Goes against integrability intuition
Prescription ideas

### Naive prescription

**LSZ with:**
1. external legs → on-shell
2. \( i\epsilon \to 0 \)

**Result:** \( \times \)

Particle production in:
- Nappi model – agrees with [Nappi, 80]
- \( S^n \) sigma model

These limits do not commute!

### ‘Off-shell’ prescription

**LSZ with swapped order of limits:**
1. \( i\epsilon \to 0 \)
2. external legs → on-shell

**Result:** ?

Watch this space!
Final comments

- Off-shell prescription requires choices: how to take on-shell limit?

- Hope to find a universal choice that manifests integrability.

- We also expect T-duality to be manifest.

- **In the future:** hope to use this S-matrix to discover new constraints imposed by integrability.