Stratifying On-Shell Cluster Varieties

Jacob L. Bourjaily

Amplitudes 2018 Summer School
QMAP, University of California, Davis

The Niels Bohr
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Organization and Outline

Amalgamating Diagrams from Three-Particle Amplitudes

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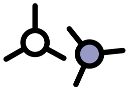

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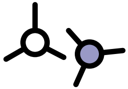

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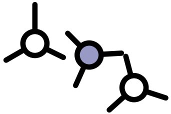

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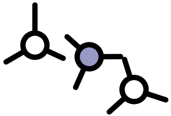

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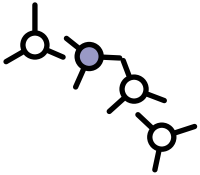

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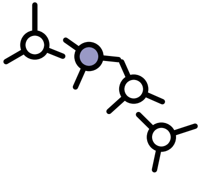

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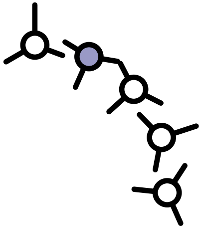

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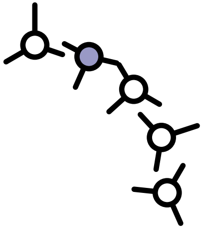

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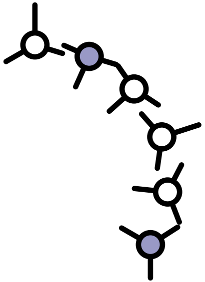

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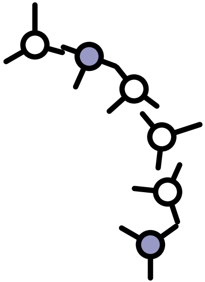

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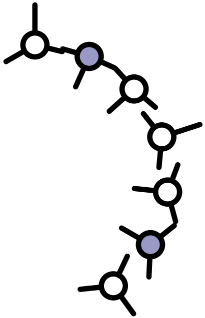

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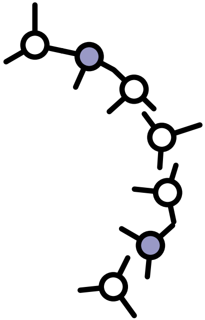

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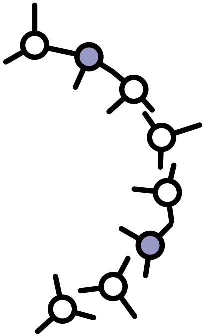

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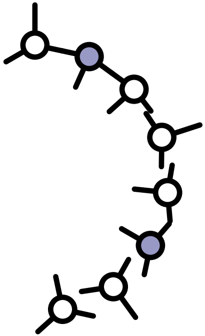

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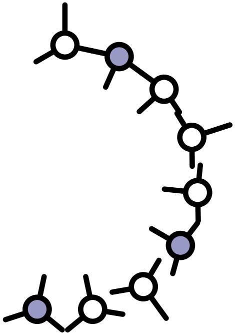

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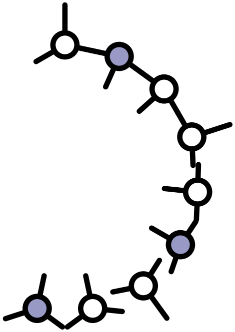

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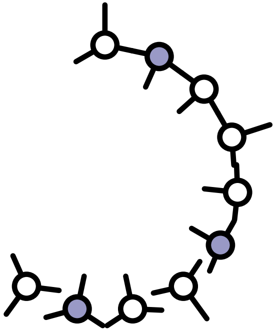

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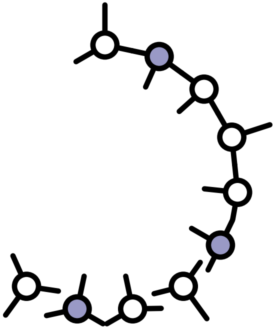

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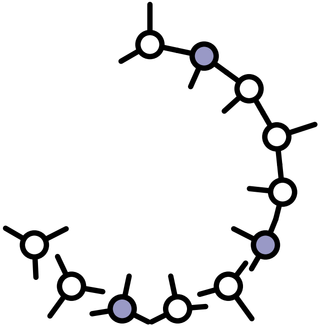

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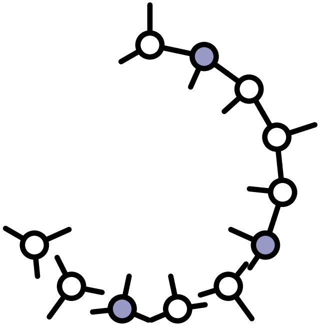

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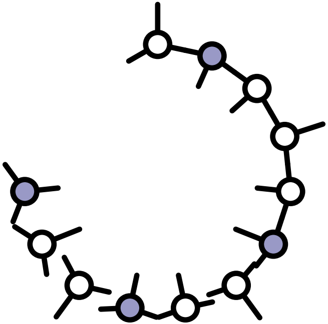

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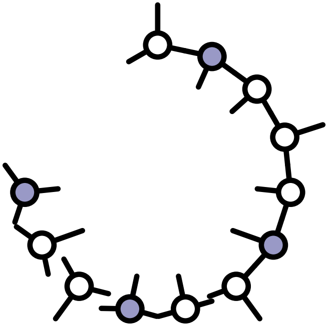

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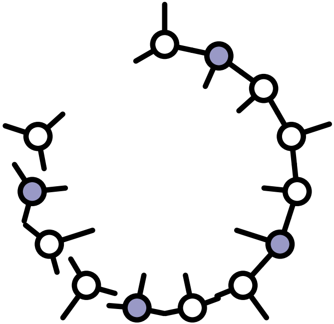

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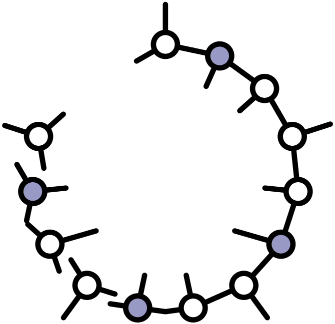

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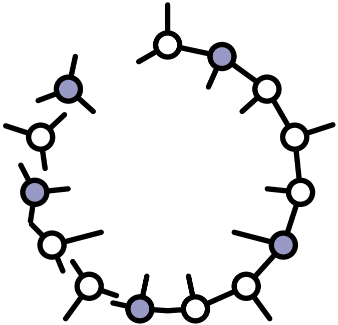

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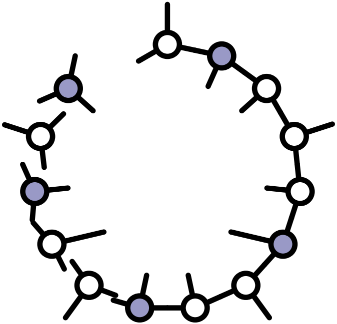

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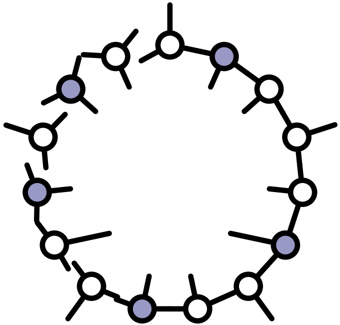

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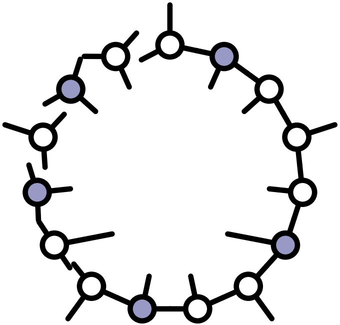

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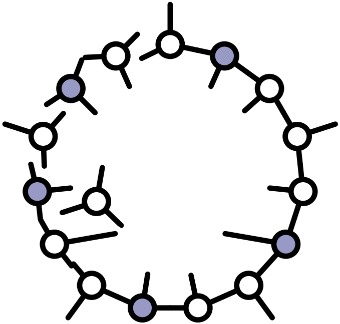

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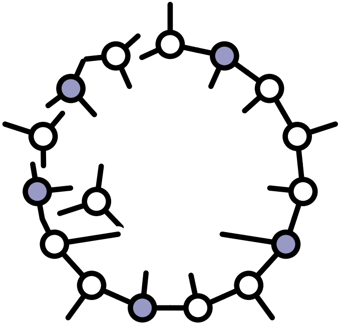

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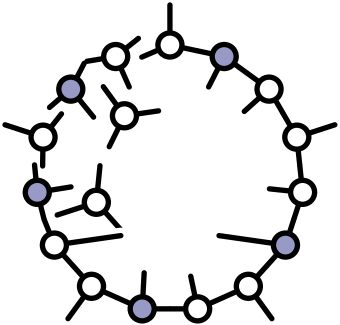

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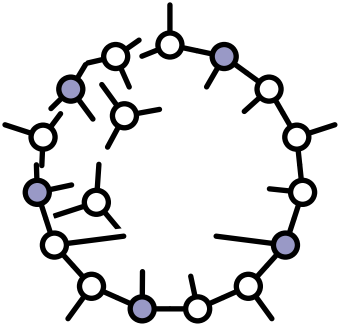

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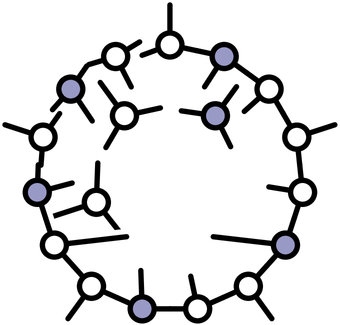

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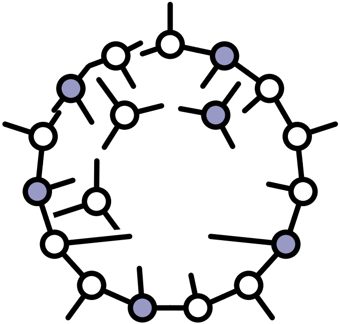

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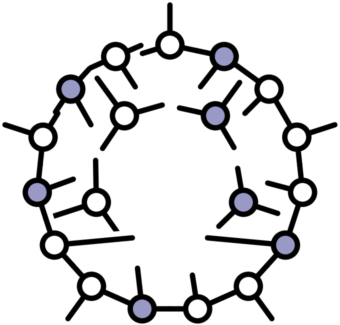

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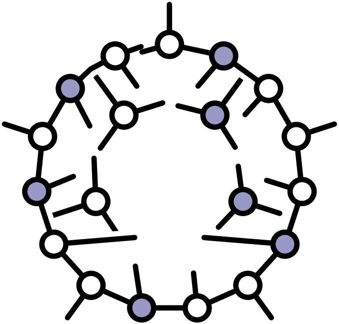

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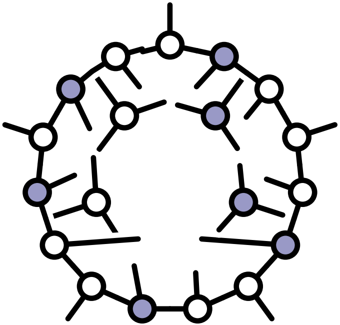

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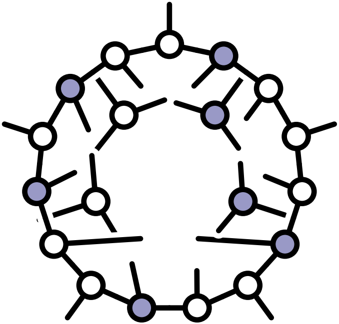

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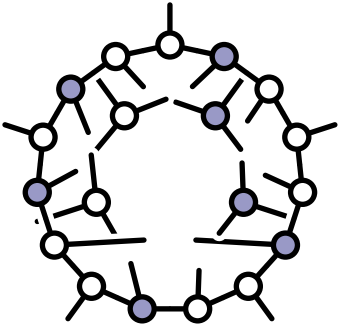

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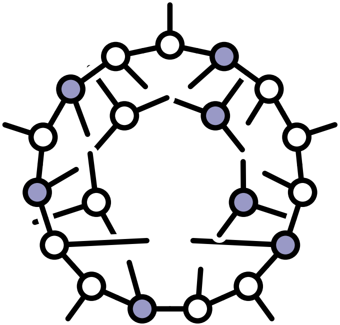

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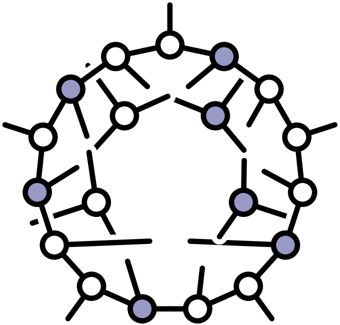

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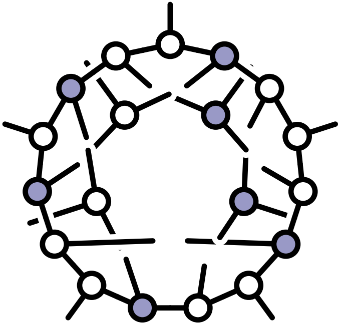

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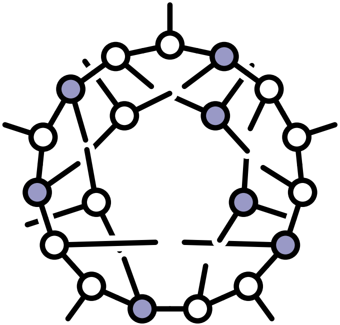

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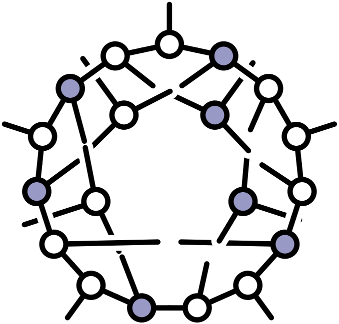

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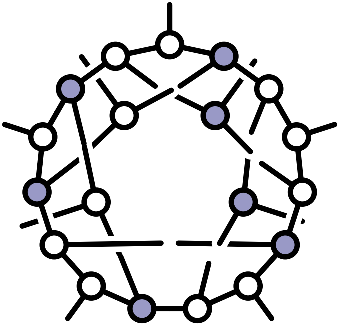

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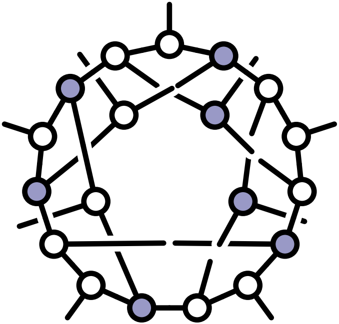

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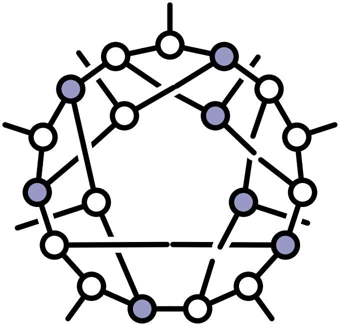

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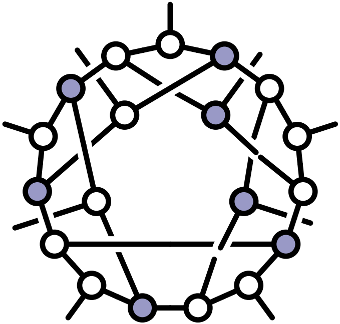

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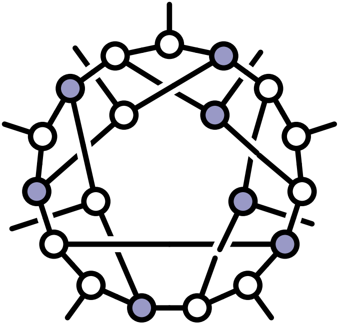

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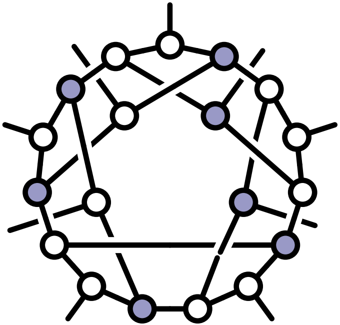

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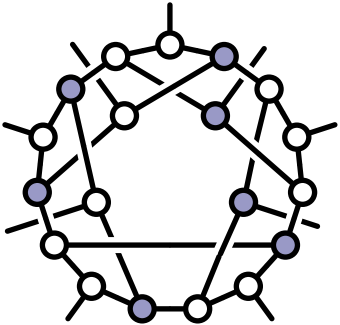

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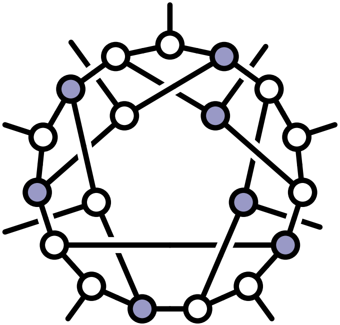

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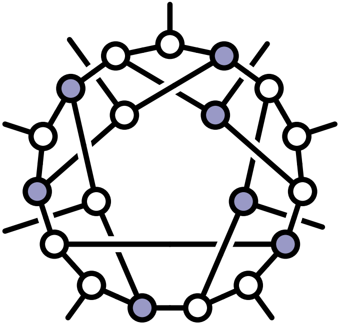

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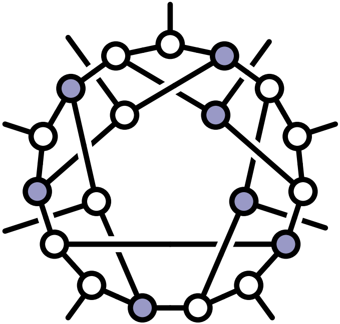

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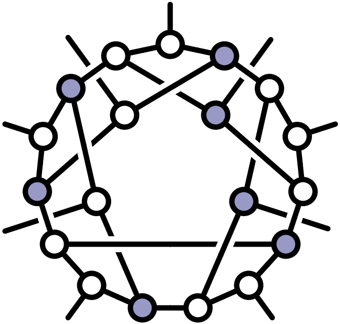

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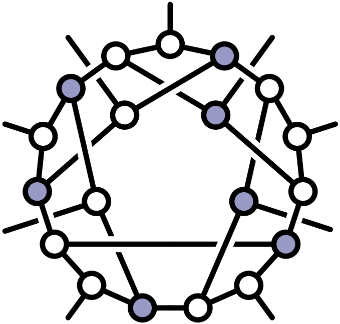

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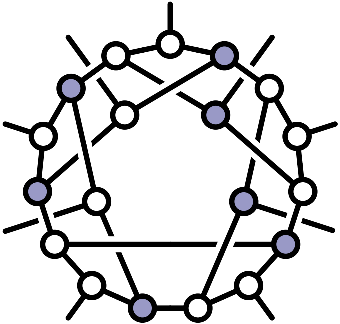

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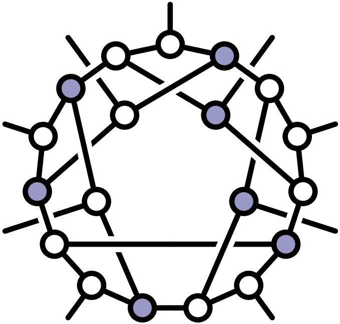

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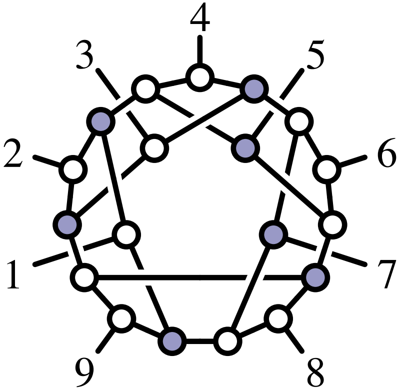

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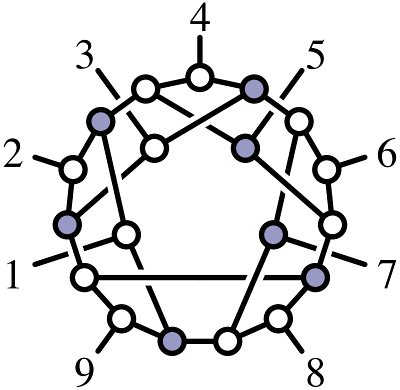

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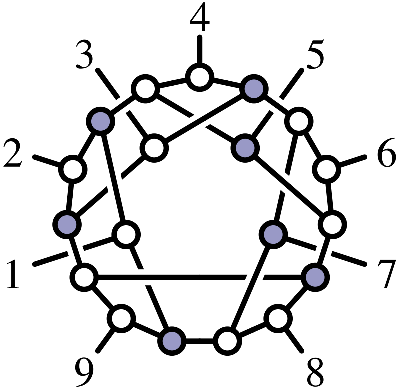

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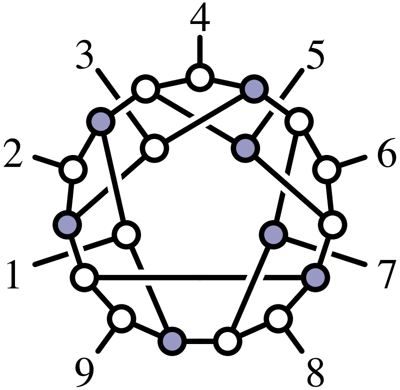

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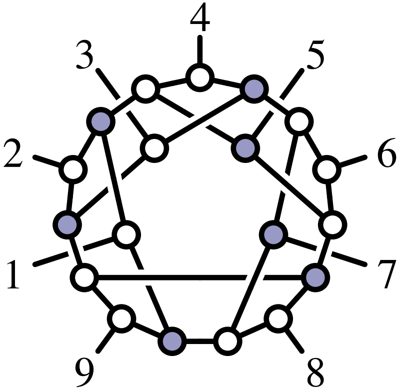

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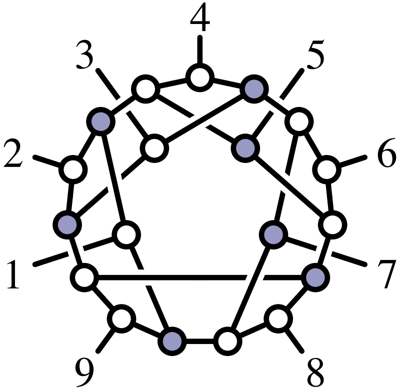

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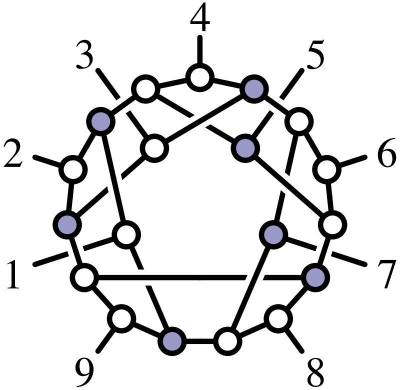

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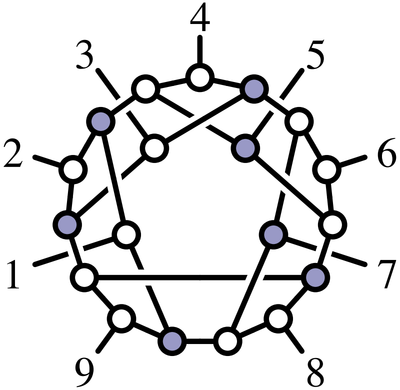

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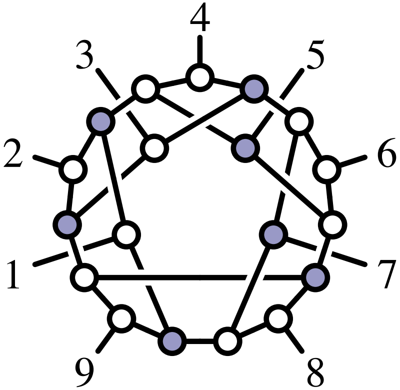

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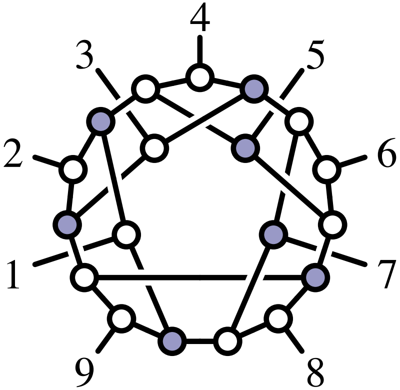

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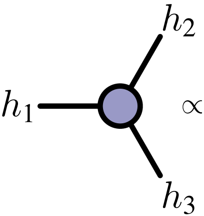
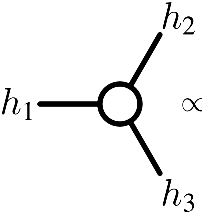
Momentum conservation and Poincaré-invariance **uniquely** fix the kinematical dependence of the amplitude for three massless particles (to all loop orders!).



$$\propto \quad \langle 12\rangle^{h_3-h_1-h_2}\langle 23\rangle^{h_1-h_2-h_3}\langle 31\rangle^{h_2-h_3-h_1}$$
$$h_1+h_2+h_3 \leq 0$$



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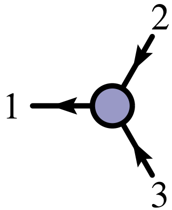

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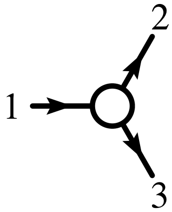

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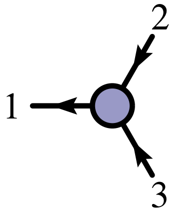

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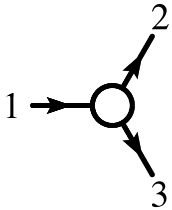

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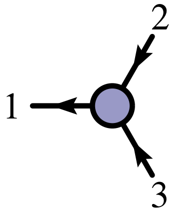

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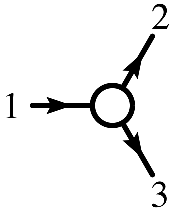

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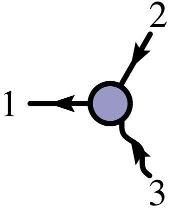

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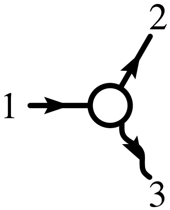

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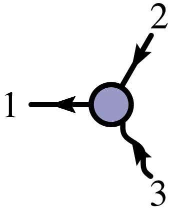

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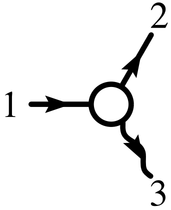

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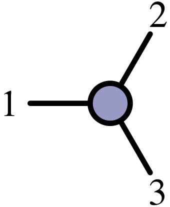
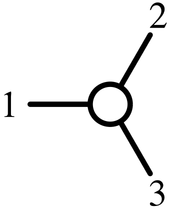


$$= \quad \frac{\delta^{2\times 4}\big(\lambda\cdot\widetilde{\eta}\big)}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,1\rangle}\;\delta^{2\times 2}\big(\lambda\cdot\widetilde{\lambda}\big) \equiv \mathcal{A}_3^{(2)}$$



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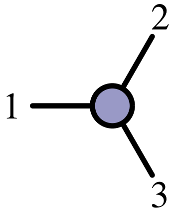

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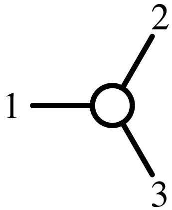

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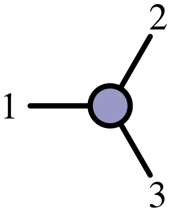

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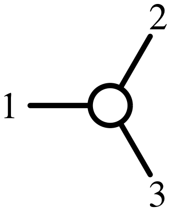

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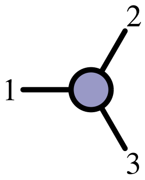

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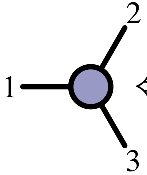

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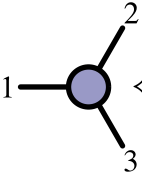

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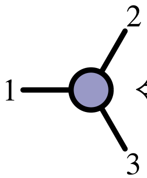

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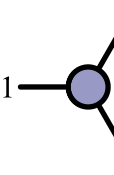
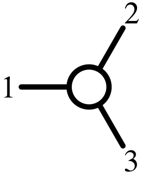


$$\Leftrightarrow B \equiv \begin{pmatrix} b_1^1 & b_2^1 & b_3^1 \\ b_1^2 & b_2^2 & b_3^2 \end{pmatrix}$$



$$\mathcal{A}_3^{(2)} = \frac{\delta^{2\times4}\left(\lambda\cdot\widetilde{\eta}\right)}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}\delta^{2\times2}\left(\lambda\cdot\widetilde{\lambda}\right) \equiv \int\frac{d^{2\times3}B}{\mathrm{vol}(GL_2)}\,\frac{\delta^{2\times4}\left(B\cdot\widetilde{\eta}\right)}{(12)(23)(31)}\;\delta^{2\times2}\left(B\cdot\widetilde{\lambda}\right)\;\delta^{1\times2}\left(\lambda\cdot B^{\perp}\right)$$

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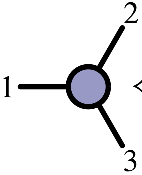

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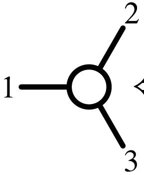

$$\Leftrightarrow W$$

$$\mathcal{A}_3^{(2)}=\frac{\delta^{2\times4}\big(\lambda\cdot\widetilde{\eta}\big)}{\langle12\rangle\langle23\rangle\langle31\rangle}\delta^{2\times2}\big(\lambda\cdot\widetilde{\lambda}\big)\;\equiv\int\frac{d^{2\times3}B}{\mathrm{vol}(GL_2)}\,\frac{\delta^{2\times4}\big(B\cdot\widetilde{\eta}\big)}{(12)(23)(31)}\;\delta^{2\times2}\big(B\cdot\widetilde{\lambda}\big)\;\delta^{1\times2}\big(\lambda\cdot B^{\perp}\big)$$

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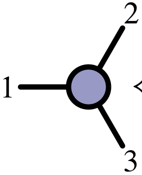

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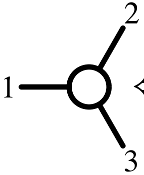

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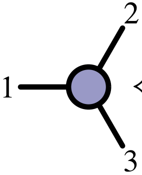

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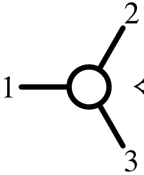

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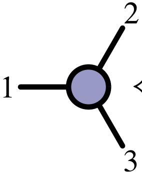

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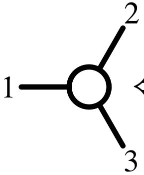

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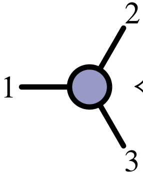
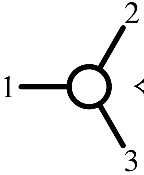


$$\Leftrightarrow B\equiv\begin{pmatrix}b_1^1 & b_2^1 & b_3^1\\ b_1^2 & b_2^2 & b_3^2\end{pmatrix}$$



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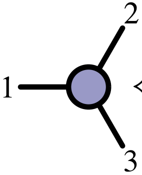

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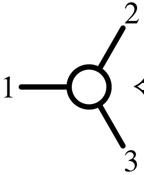

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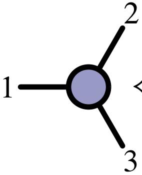

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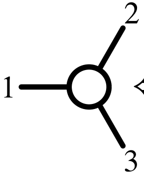

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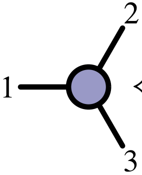

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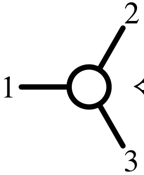

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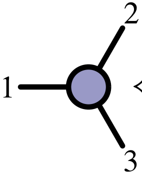
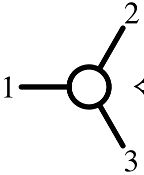


$$\Leftrightarrow B\equiv\begin{pmatrix}1 & 0 & b_3^1\\ 0 & 1 & b_3^2\end{pmatrix}$$



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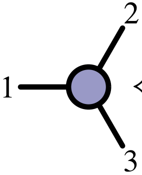
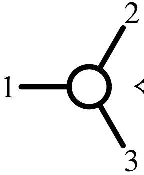


$$\Leftrightarrow B \equiv \begin{pmatrix} b_1^1 & 1 & 0 \\ b_1^2 & 0 & 1 \end{pmatrix}$$



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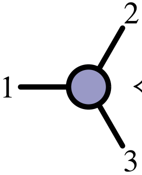
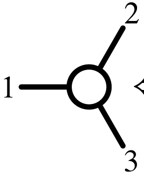


$$\Leftrightarrow B \equiv \begin{pmatrix} 0 & b_2^1 & 1 \\ 1 & b_2^2 & 0 \end{pmatrix}$$



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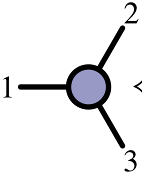

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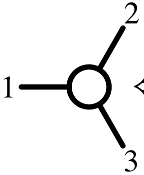

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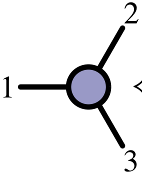

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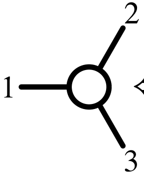

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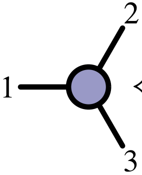

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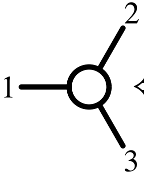

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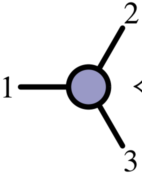

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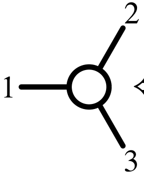

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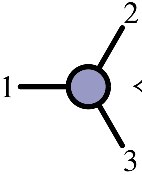

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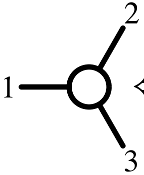

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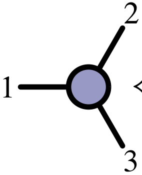

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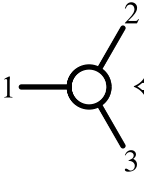

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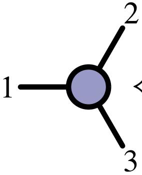

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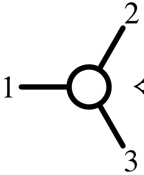

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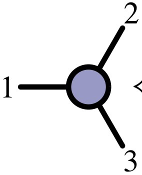

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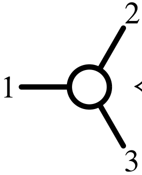

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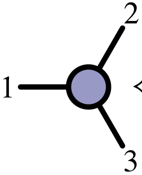

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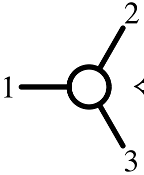

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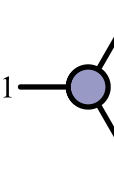

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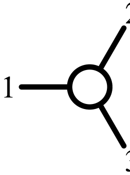

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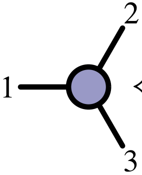

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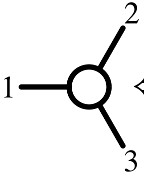

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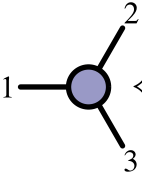

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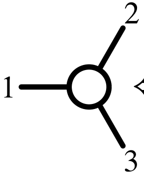

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Constructing the Correspondence: Amalgamations & Bridges

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Direct/Outer Products

$$(f_1,f_2)\mapsto\; f_1\times f_2$$
$$(C_1,C_2)\mapsto C_1\oplus C_2\subset G(k_1{+}k_2,n_1{+}n_2)$$
$$(\Omega_1,\Omega_2)\mapsto\Omega_1\wedge\Omega_2\quad(d_1,d_2)\mapsto d_1{+}d_2$$

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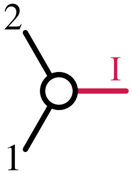
$$(\Omega_1, \Omega_2) \mapsto \Omega_1 \wedge \Omega_2 \quad (d_1, d_2) \mapsto d_1 + d_2$$



Constructing the Correspondence: Amalgamations & Bridges

Direct/Outer Products

$$(f_1, f_2) \mapsto f_1 \times f_2$$
$$(C_1, C_2) \mapsto C_1 \oplus C_2 \subset G(k_1+k_2, n_1+n_2)$$
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$$\begin{array}{c} \begin{array}{ccc} 1 & 2 & \mathrm{I} \end{array} \\ \left(\begin{array}{ccc} 1 & w_2 & w_{\mathrm{I}} \end{array}\right) \end{array}$$

Constructing the Correspondence: Amalgamations & Bridges

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$$(f_1,f_2)\mapsto f_1\times f_2$$
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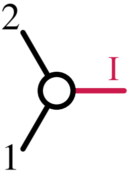
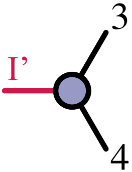


$$\begin{array}{ccc} 1 & 2 & \mathrm{I} \\ \hline \end{array}$$
$$\begin{pmatrix} 1 & w_2 & w_{\mathrm{I}} \end{pmatrix}$$



$$\begin{array}{ccc} \mathrm{I}' & 3 & 4 \\ \hline \end{array}$$
$$\begin{pmatrix} 1 & 0 & b_4^1 \\ 0 & 1 & b_4^2 \end{pmatrix}$$

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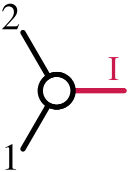
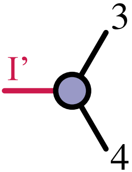


$$\begin{array}{c}\begin{array}{ccc}1 & 2 & \mathrm{I}\end{array}\\ \begin{pmatrix}1 & w_2 & w_{\mathrm{I}}\end{pmatrix}\end{array}$$



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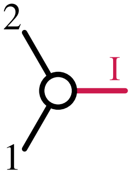

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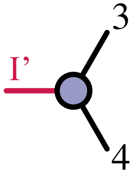

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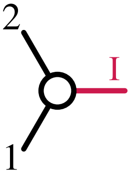

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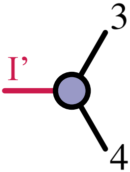

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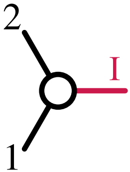
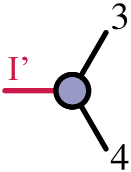


$$\begin{array}{ccc} 1 & 2 & \mathrm{I} \\ \hline \multicolumn{3}{c}{\begin{pmatrix} 1 & w_2 & w_{\mathrm{I}} \end{pmatrix}} \end{array}$$



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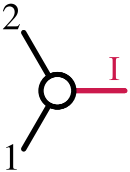

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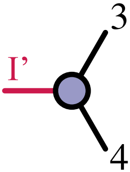

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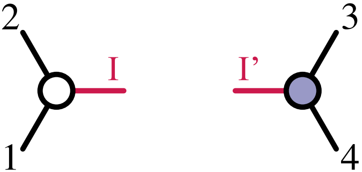

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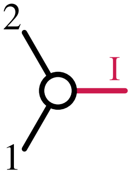

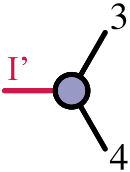

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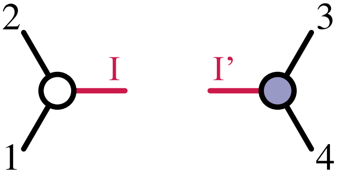

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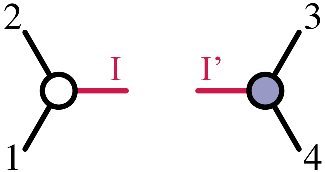

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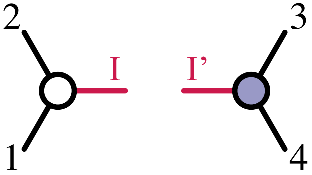

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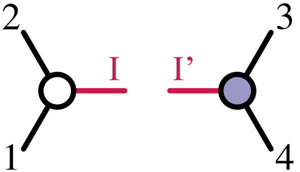

$$\begin{array}{ccc} 1 & 2 & \mathrm{I} \\ \hline \end{array} \qquad \begin{array}{ccc} \mathrm{I'} & 3 & 4 \\ \hline \end{array}$$

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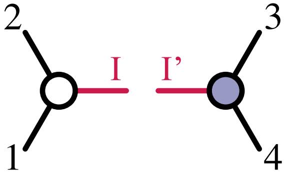

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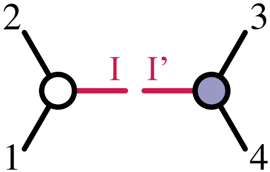

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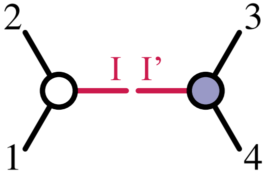

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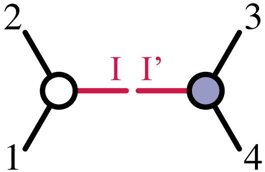

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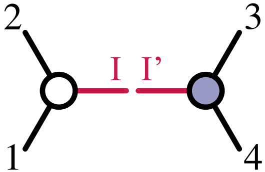

$$C \equiv \begin{array}{c} \begin{array}{cccccc} 1 & 2 & \mathrm{I} & \mathrm{I'} & 3 & 4 \end{array} \\ \left(\begin{array}{cccccc} 1 & w_2 & w_{\mathrm{I}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & b_4^1 \\ 0 & 0 & 0 & 0 & 1 & b_4^2 \end{array}\right) \end{array}$$

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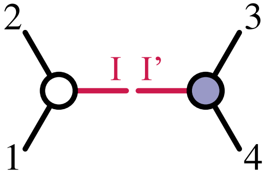

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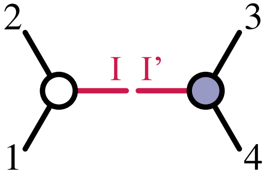

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Amalgamation: Gluing Legs (A,B)

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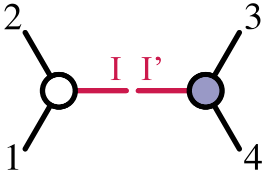

$$C\equiv\begin{array}{c}\begin{array}{cccccc}1&2&\mathrm{I}&\mathrm{I'}&3&4\end{array}\\\left(\begin{array}{cccccc}1&w_2&w_{\mathrm{I}}&0&0&0\\0&0&0&1&0&b_4^1\\0&0&0&0&1&b_4^2\end{array}\right)\end{array}$$

Constructing the Correspondence: Amalgamations & Bridges

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$$(f_1,f_2) \mapsto f_1 \times f_2$$
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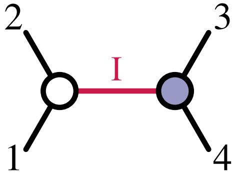

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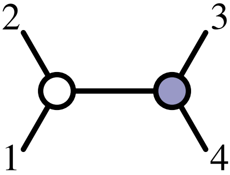

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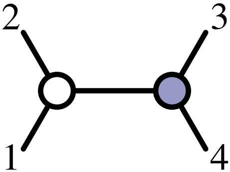

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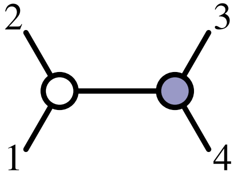

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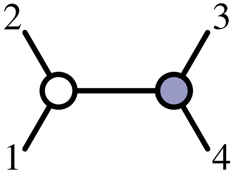

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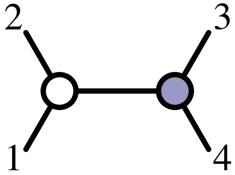

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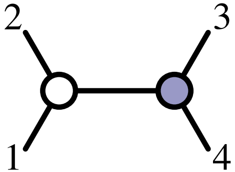

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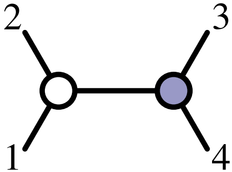

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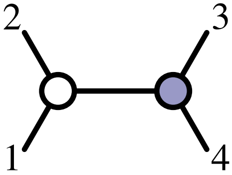

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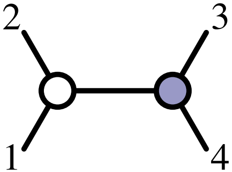

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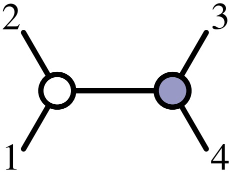

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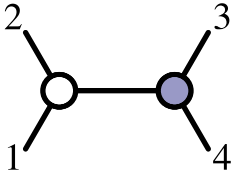

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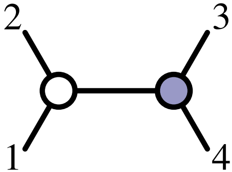

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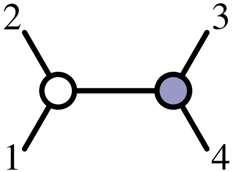

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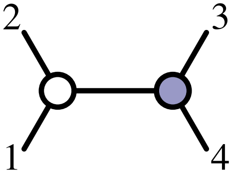

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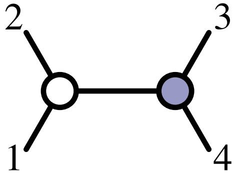

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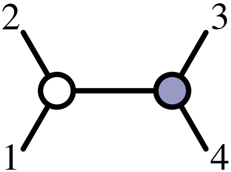

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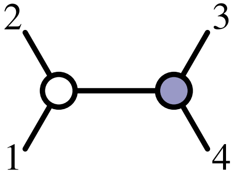

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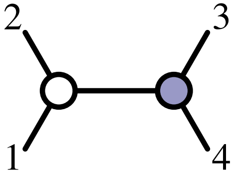

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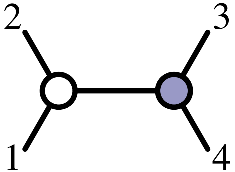

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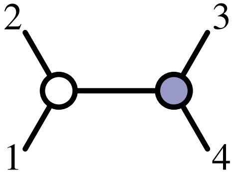

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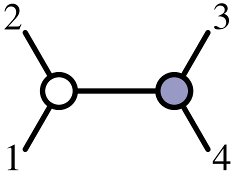

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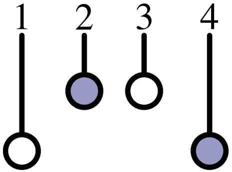

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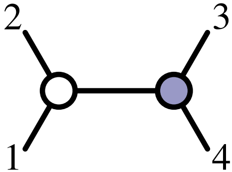

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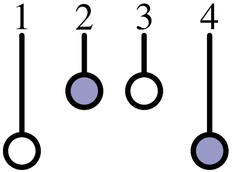

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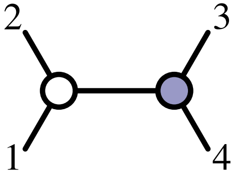

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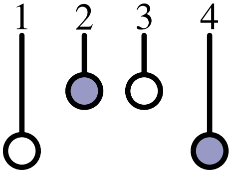

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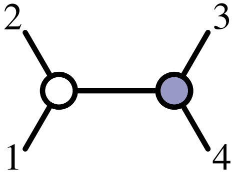

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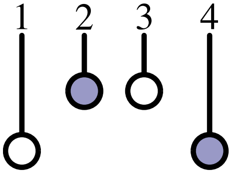

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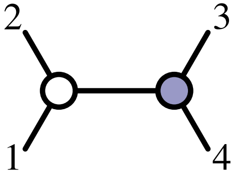

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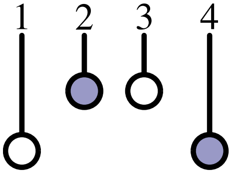

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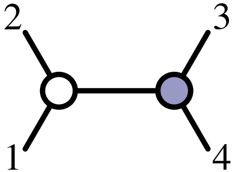

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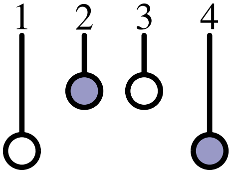

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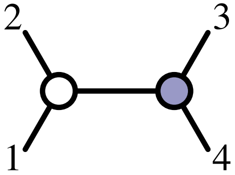

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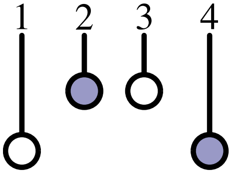

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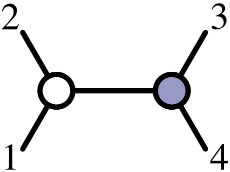

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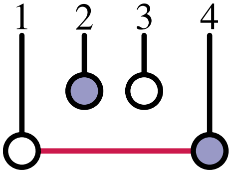

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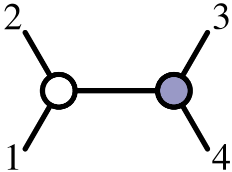

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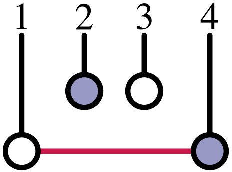

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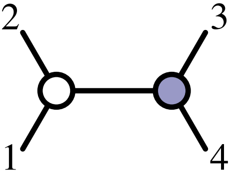

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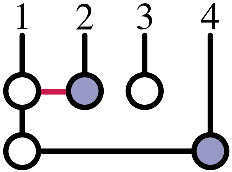

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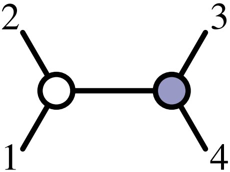

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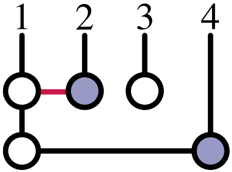

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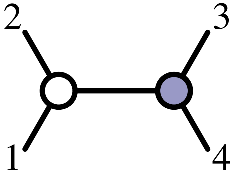

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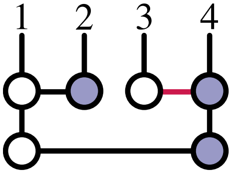

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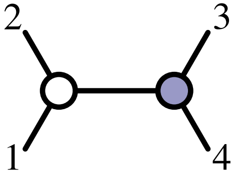

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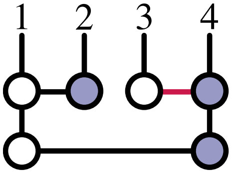

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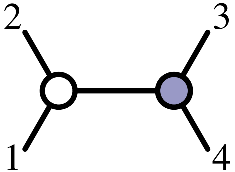

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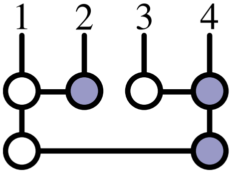

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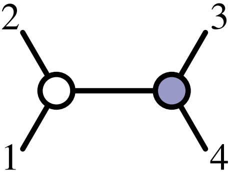

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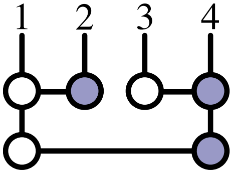

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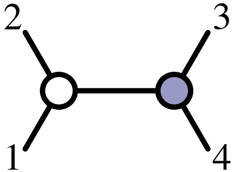

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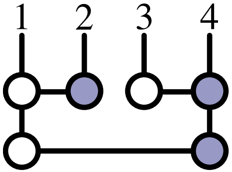

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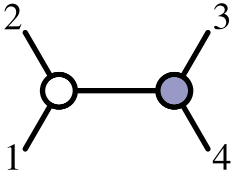

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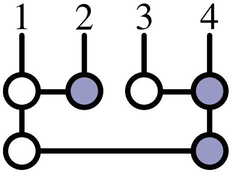

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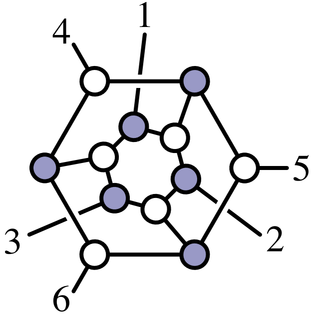

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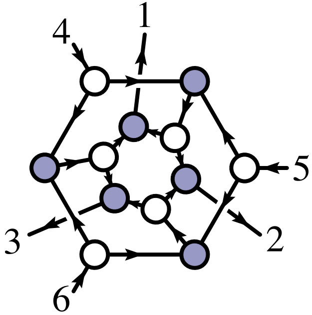

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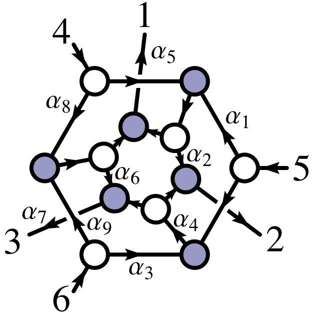

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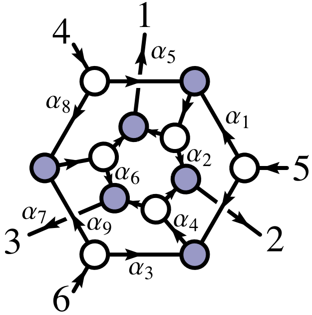

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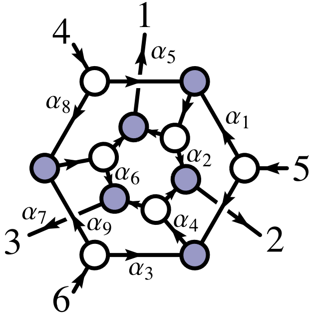

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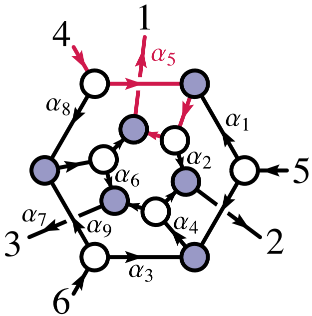

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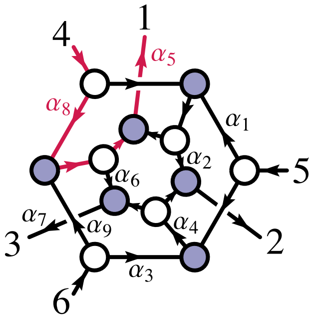

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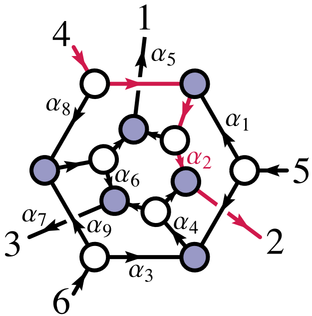

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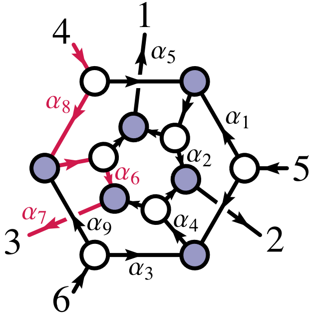

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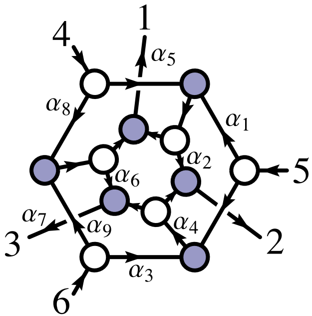

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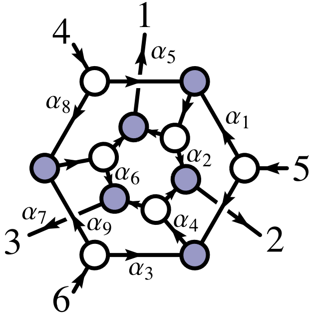

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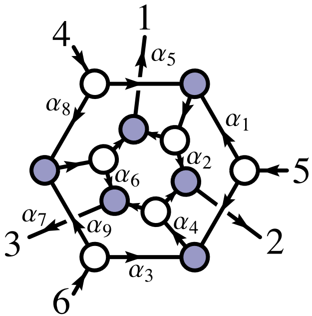

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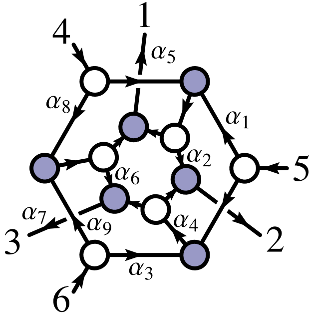

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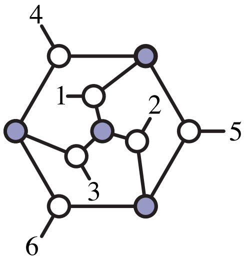

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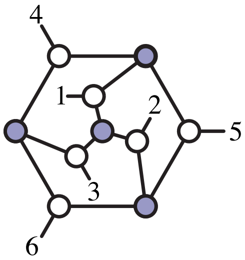

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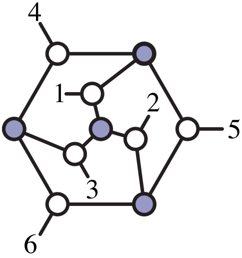

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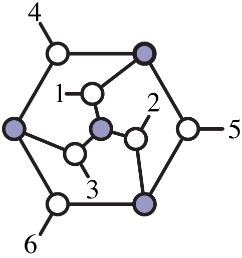

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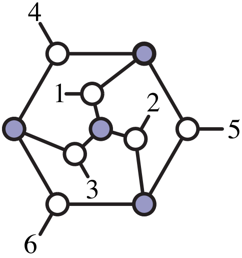

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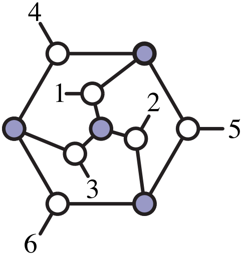

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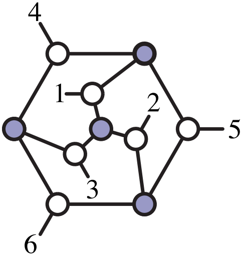

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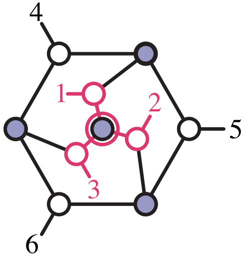

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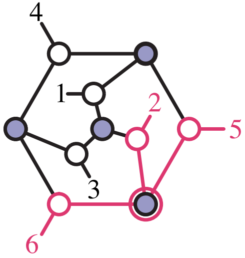

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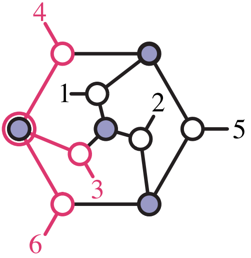

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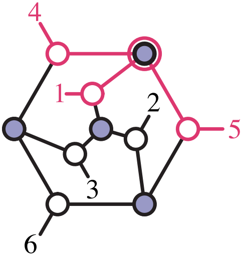

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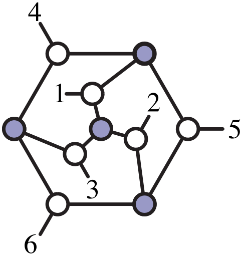

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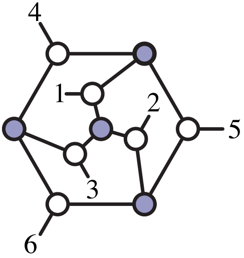

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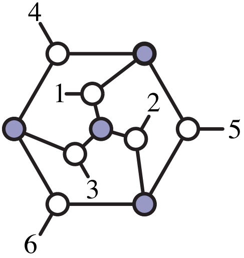

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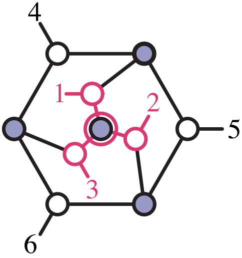

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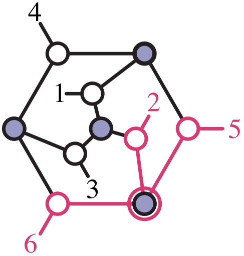

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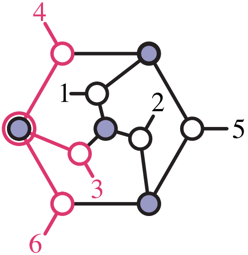

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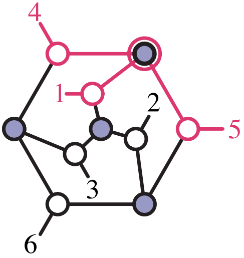

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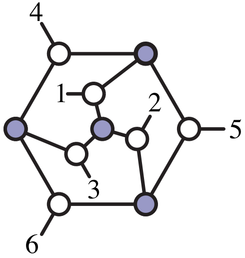

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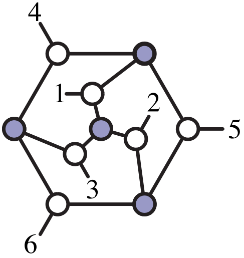

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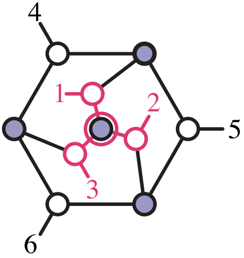

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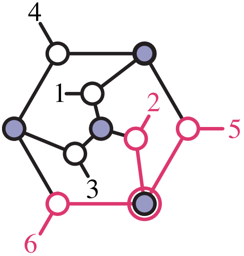

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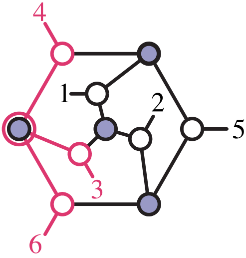

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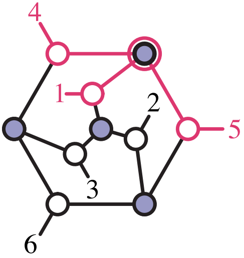

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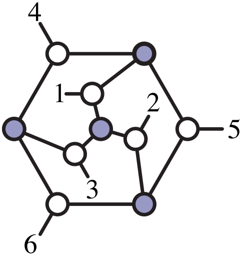

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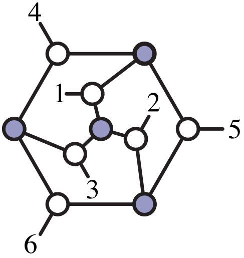

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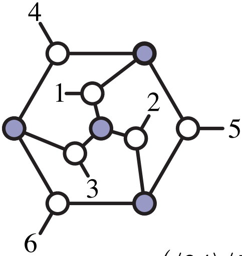

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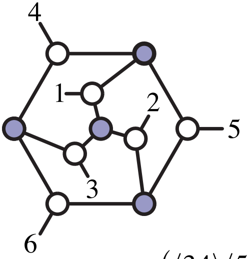

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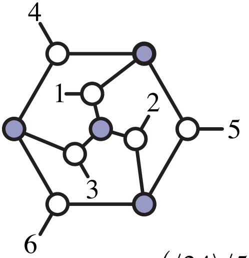

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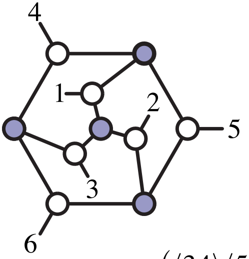

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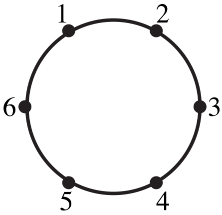

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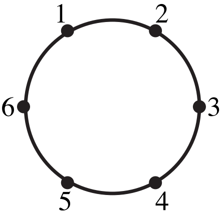

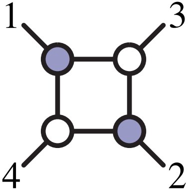

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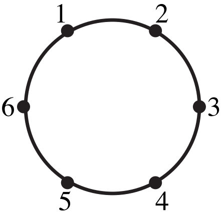

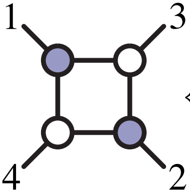

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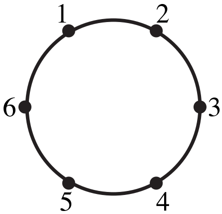

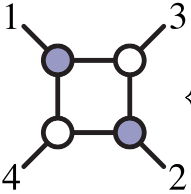

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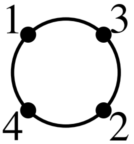

PT(1, 3, 2, 4)

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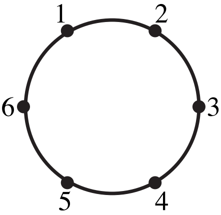

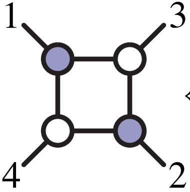

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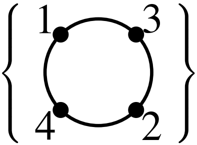

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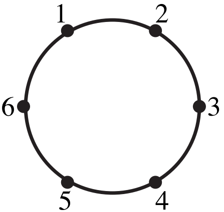

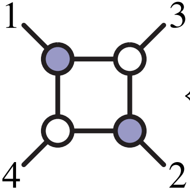

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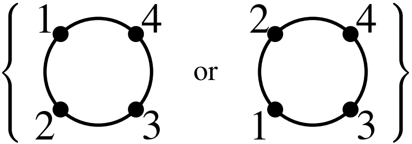

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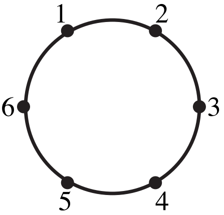

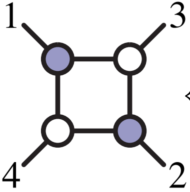

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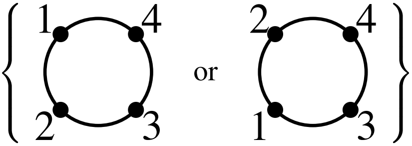

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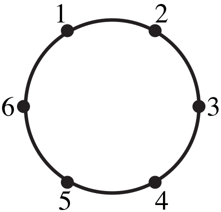

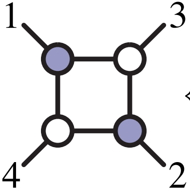

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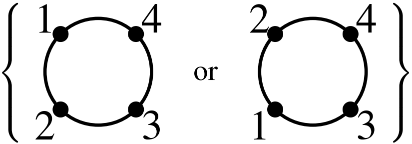

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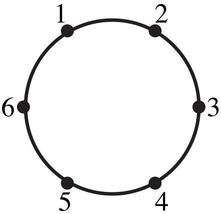

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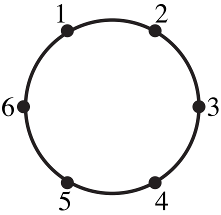

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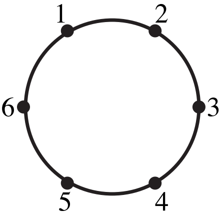

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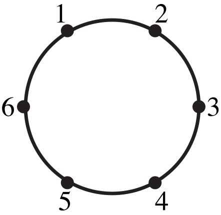

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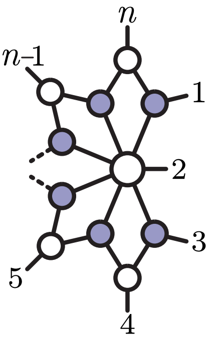

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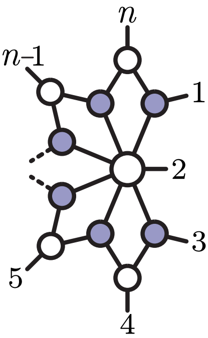

$$\left\{\begin{matrix} (2 & n & 1) \\ (2 & 3 & 4) \\ (2 & 4 & 5) \\ & \vdots & \\ (2 & n-1 & n) \end{matrix}\right\}$$

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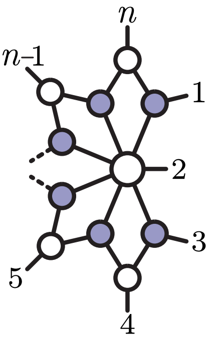

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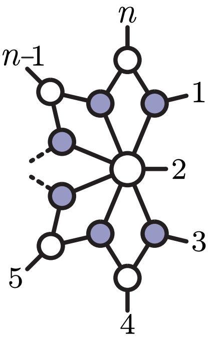

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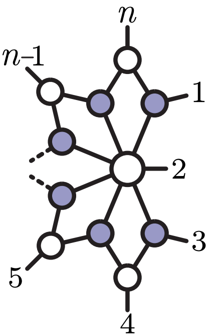

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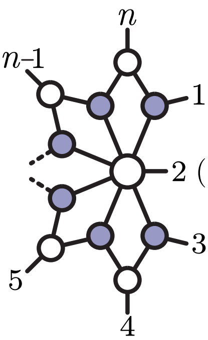

$$(-1)\frac{\left\{\begin{matrix}(2 & n & 1)\\ (2 & 3 & 4)\\ (2 & 4 & 5)\\ & \vdots & \\ (2 & n{-}1 & n)\end{matrix}\right\}}{\mathrm{PT}(1,2,\ldots,n)} = \frac{\left\{\begin{matrix}(2 & 1 & n)\\ (2 & 3 & 4)\\ (2 & 4 & 5)\\ & \vdots & \\ (2 & n{-}1 & n)\end{matrix}\right\}}{\mathrm{PT}(1,n,2,\ldots,n{-}1)+\ldots+\mathrm{PT}(1,3,\ldots,n,2)}$$

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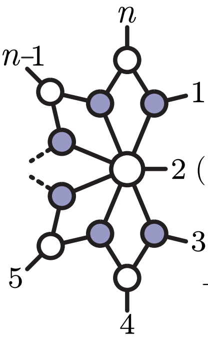

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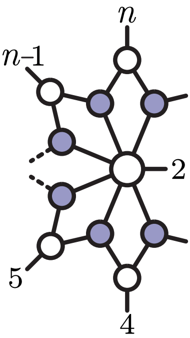

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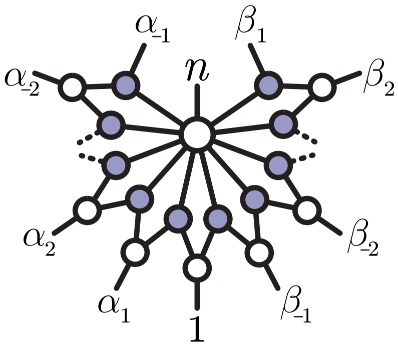

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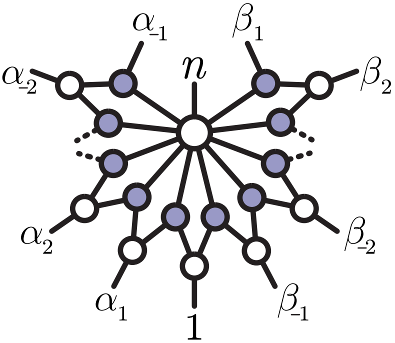

$$\left\{\begin{array}{c}(1\ \alpha_1\ n)\\(\alpha_1\ \alpha_2\ n)\\ \vdots \\(\alpha_{-2}\ \alpha_{-1}\ n)\\(n\ \beta_1\ \beta_2)\\ \vdots \\(n\ \beta_{-2}\ \beta_{-1})\\(n\ \beta_{-1}\ 1)\end{array}\right\}$$

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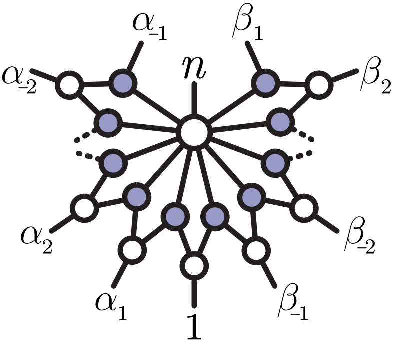

$$\left\{\begin{pmatrix} (\;1 & \alpha_1 & n\;) \\ (\alpha_1 & \alpha_2 & n\;) \\ & \vdots & \\ (\alpha_{-2} & \alpha_{-1} & n\;) \\ (\;n & \beta_1 & \beta_2) \\ & \vdots & \\ (\;n & \beta_{-2} & \beta_{-1}) \\ (\;n & \beta_{-1} & 1\;) \end{pmatrix}\right\} \qquad \left\{\begin{pmatrix} (\;1 & \alpha_1 & n\;) \\ (\alpha_1 & \alpha_2 & n\;) \\ & \vdots & \\ (\alpha_{-2} & \alpha_{-1} & n\;) \\ (\;n & \beta_2 & \beta_1) \\ & \vdots & \\ (\;n & \beta_{-1} & \beta_{-2}) \\ (\;n & 1 & \beta_{-1}) \end{pmatrix}\right\}$$

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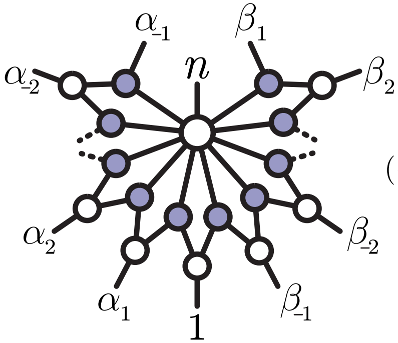

$$(-1)^{n_\beta}\left\{\begin{pmatrix} (\,1 & \alpha_1 & n\,) \\ (\alpha_1 & \alpha_2 & n\,) \\ & \vdots & \\ (\alpha_{-2} & \alpha_{-1} & n\,) \\ (\,n & \beta_1 & \beta_2) \\ & \vdots & \\ (\,n & \beta_{-2} & \beta_{-1}) \\ (\,n & \beta_{-1} & 1\,) \end{pmatrix}\right\} = \left\{\begin{pmatrix} (\,1 & \alpha_1 & n\,) \\ (\alpha_1 & \alpha_2 & n\,) \\ & \vdots & \\ (\alpha_{-2} & \alpha_{-1} & n\,) \\ (\,n & \beta_2 & \beta_1) \\ & \vdots & \\ (\,n & \beta_{-1} & \beta_{-2}) \\ (\,n & 1 & \beta_{-1}) \end{pmatrix}\right\}$$

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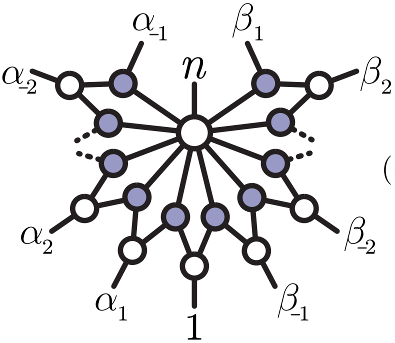

$$(-1)^{n_\beta}\begin{Bmatrix} (1\ \alpha_1\ n) \\ (\alpha_1\ \alpha_2\ n) \\ \vdots \\ (\alpha_{-2}\ \alpha_{-1}\ n) \\ (n\ \beta_1\ \beta_2) \\ \vdots \\ (n\ \beta_{-2}\ \beta_{-1}) \\ (n\ \beta_{-1}\ 1) \end{Bmatrix} = \begin{Bmatrix} (1\ \alpha_1\ n) \\ (\alpha_1\ \alpha_2\ n) \\ \vdots \\ (\alpha_{-2}\ \alpha_{-1}\ n) \\ (n\ \beta_2\ \beta_1) \\ \vdots \\ (n\ \beta_{-1}\ \beta_{-2}) \\ (n\ 1\ \beta_{-1}) \end{Bmatrix}$$

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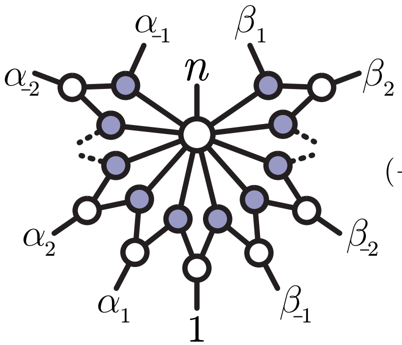

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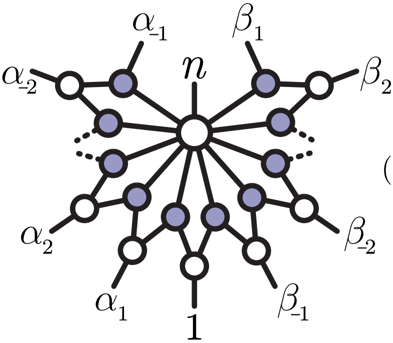

$$(-1)^{n_\beta}\left\{\begin{array}{c}(1\ \alpha_1\ n)\\(\alpha_1\ \alpha_2\ n)\\ \vdots \\(\alpha_{-2}\ \alpha_{-1}\ n)\\(n\ \beta_1\ \beta_2)\\ \vdots \\(n\ \beta_{-2}\ \beta_{-1})\\(n\ \beta_{-1}\ 1)\end{array}\right\} = \left\{\begin{array}{c}(1\ \alpha_1\ n)\\(\alpha_1\ \alpha_2\ n)\\ \vdots \\(\alpha_{-2}\ \alpha_{-1}\ n)\\(n\ \beta_2\ \beta_1)\\ \vdots \\(n\ \beta_{-1}\ \beta_{-2})\\(n\ 1\ \beta_{-1})\end{array}\right\}$$

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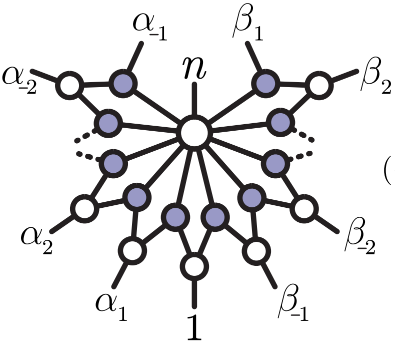

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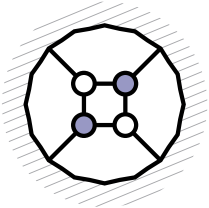

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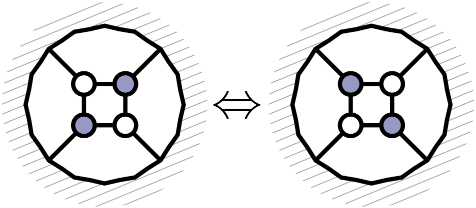

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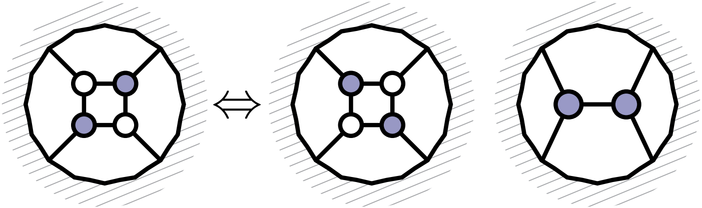

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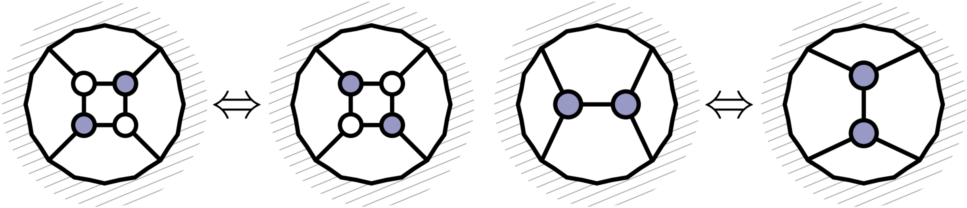

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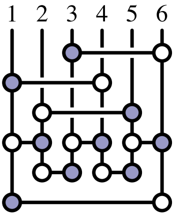

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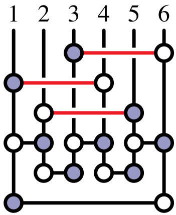

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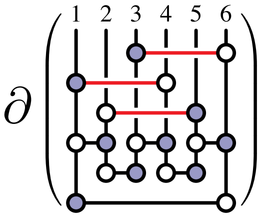

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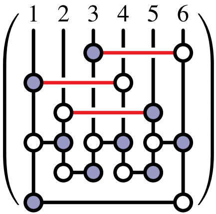

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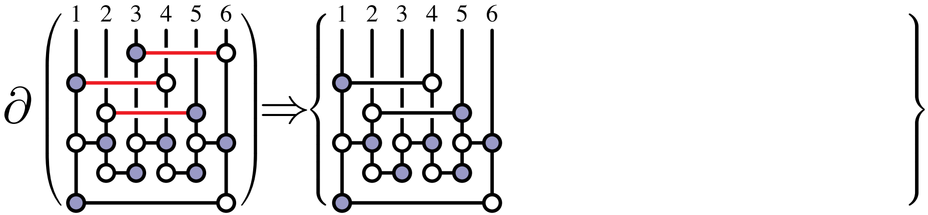

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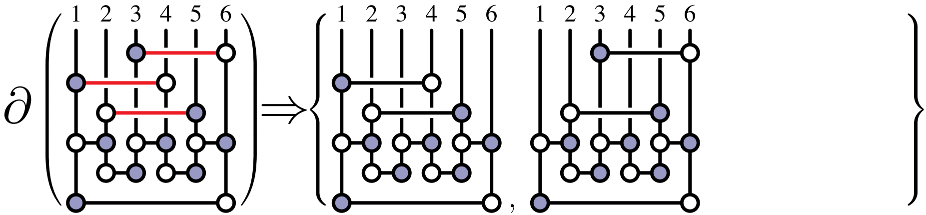

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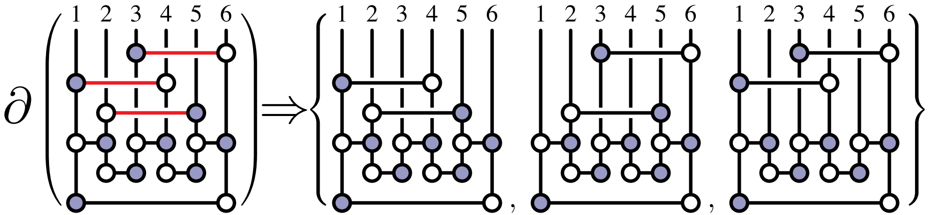

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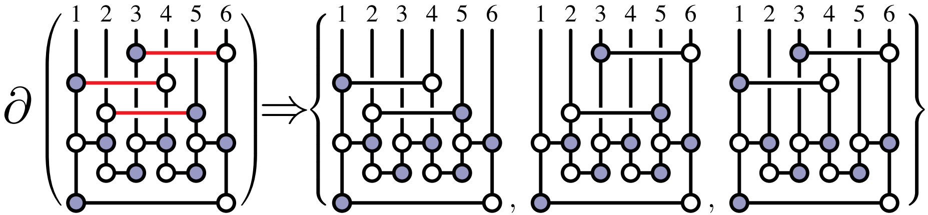

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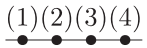

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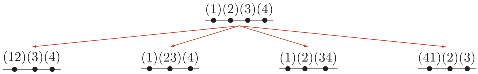

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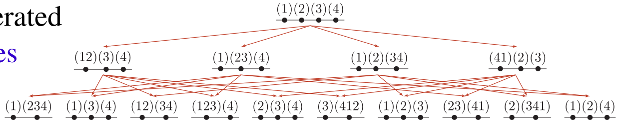

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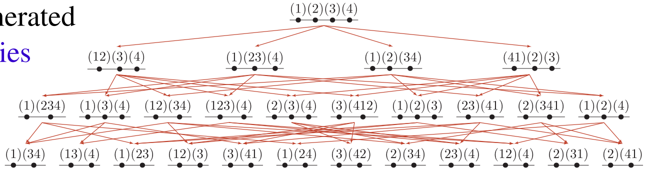

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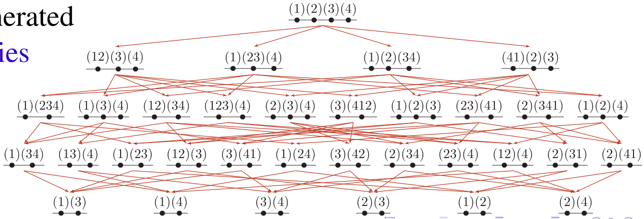

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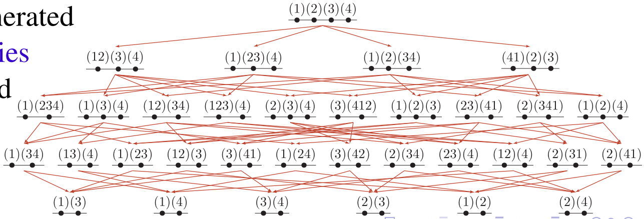

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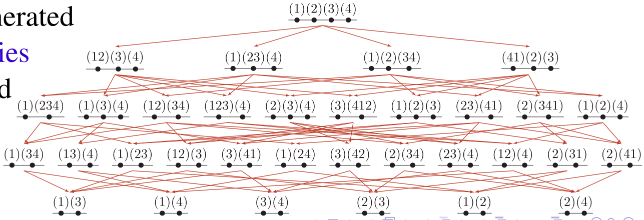

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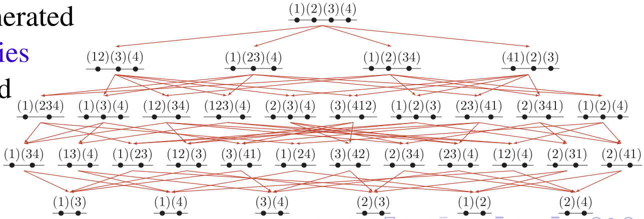

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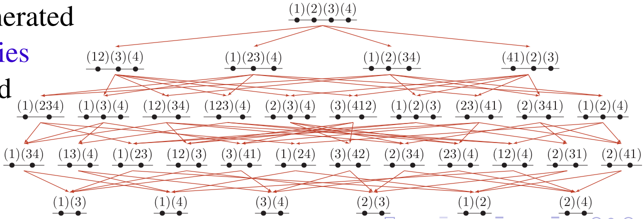

Summary of the Classification of On-Shell Varieties of $G(3,6)$

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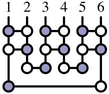

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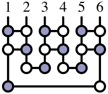

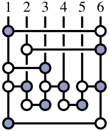

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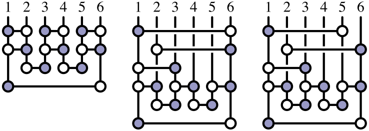

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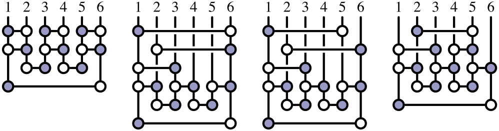

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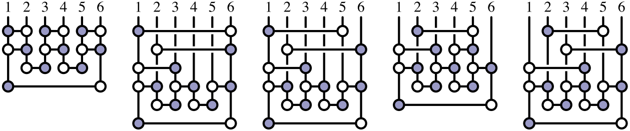

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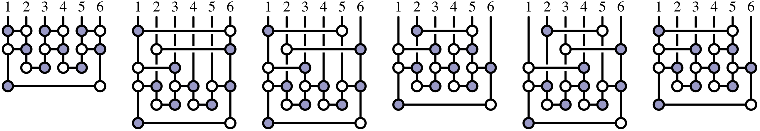

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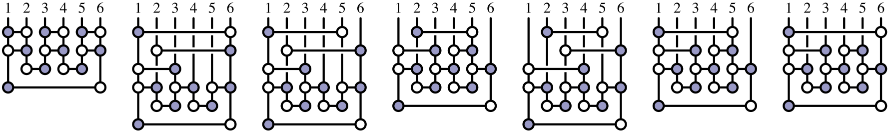

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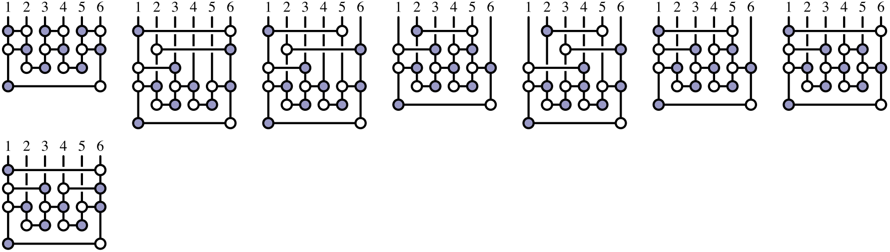

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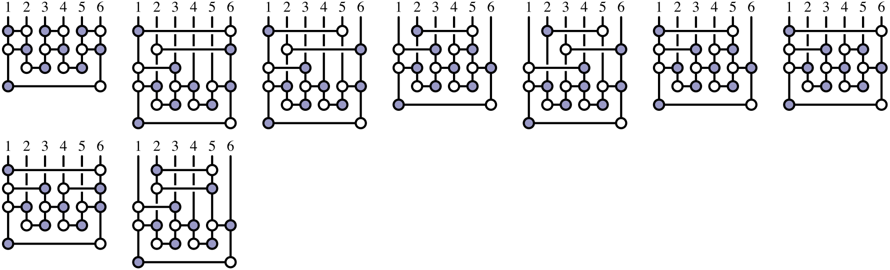

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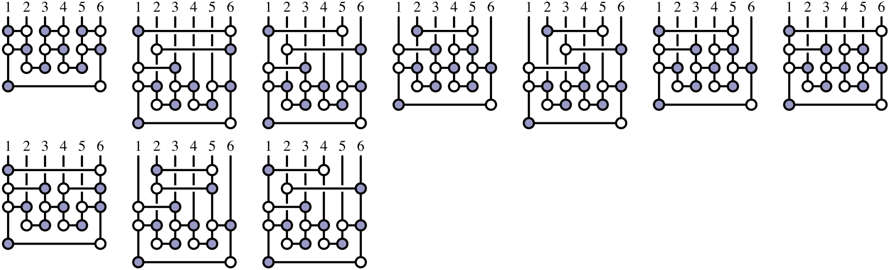

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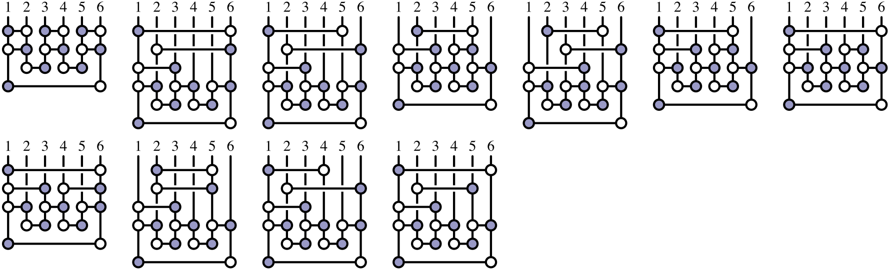

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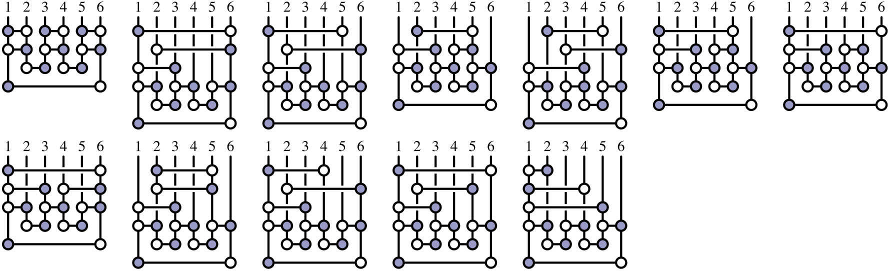

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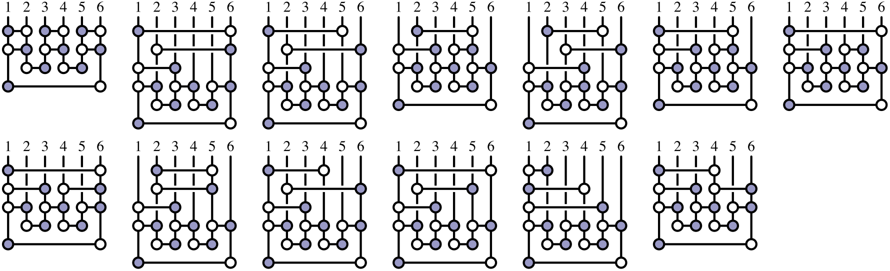

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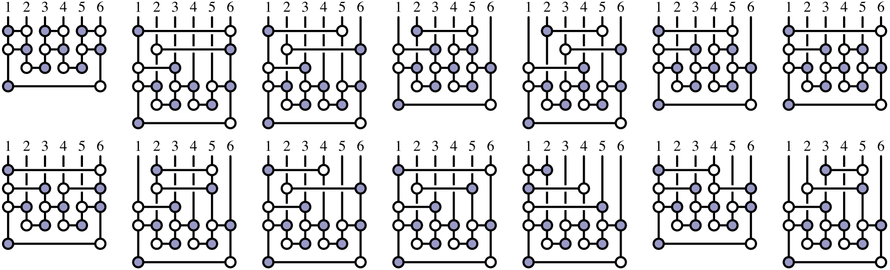

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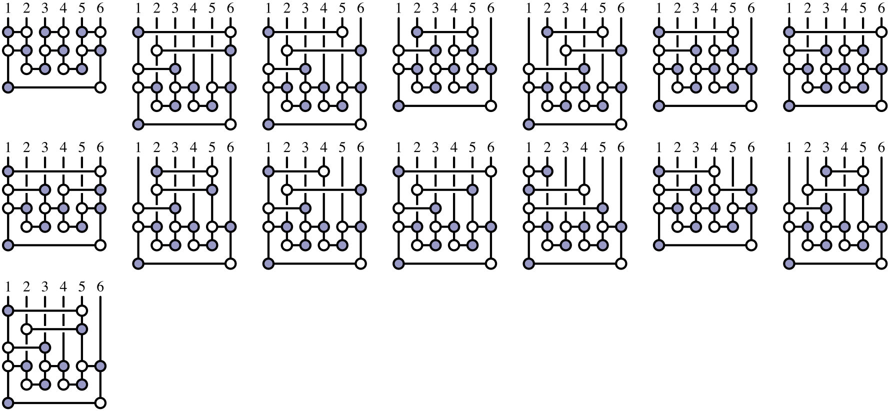

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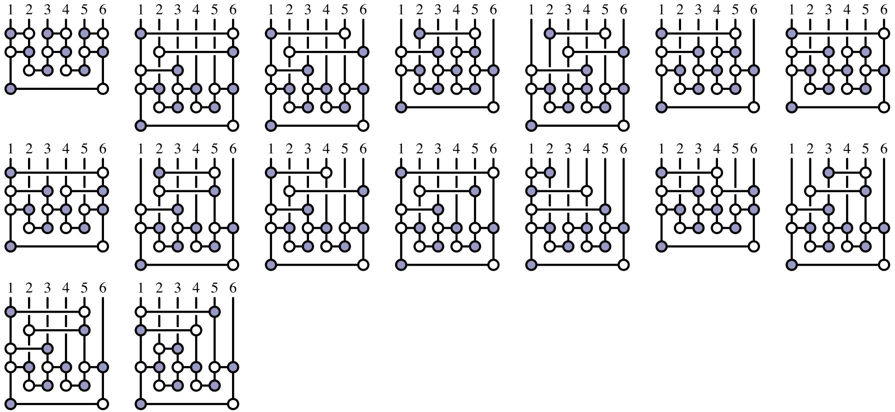

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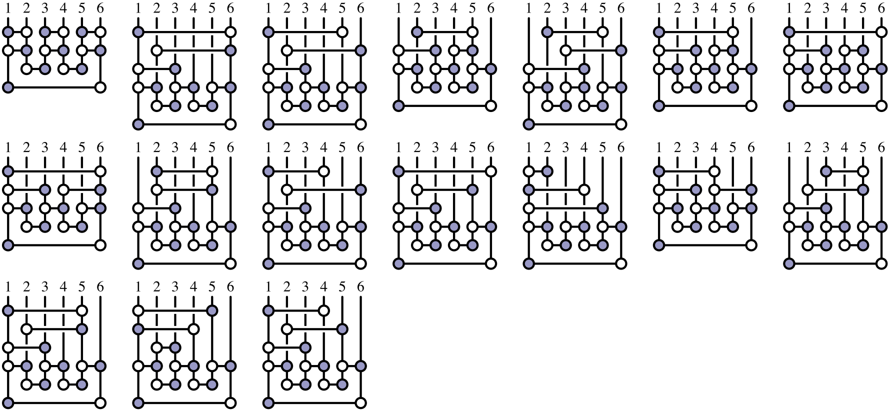

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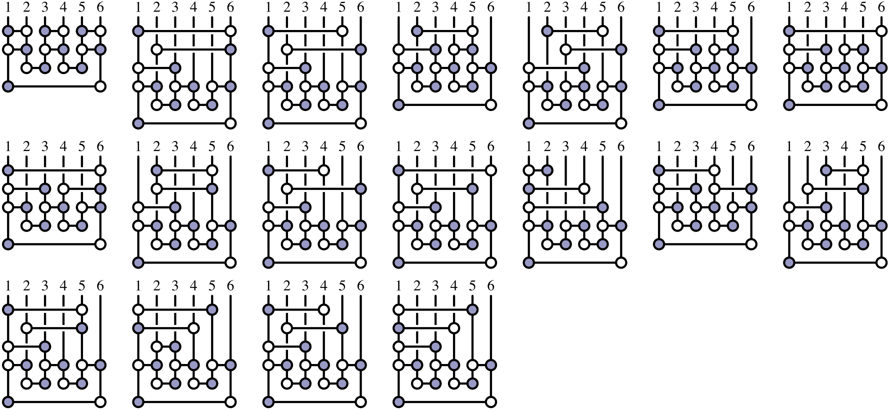

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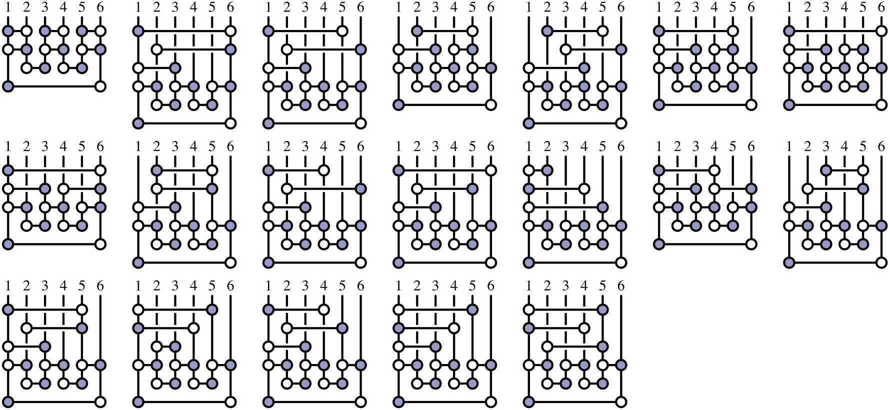

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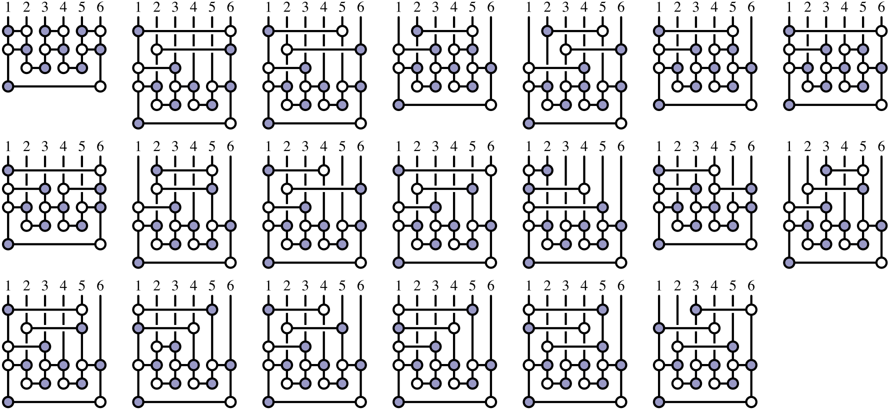

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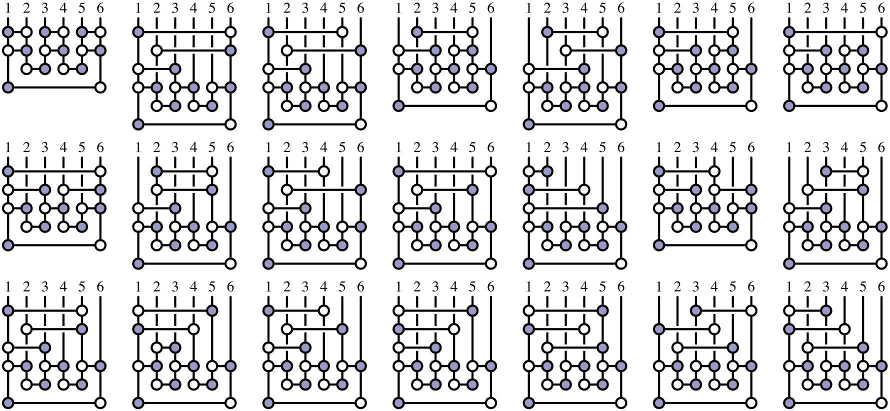

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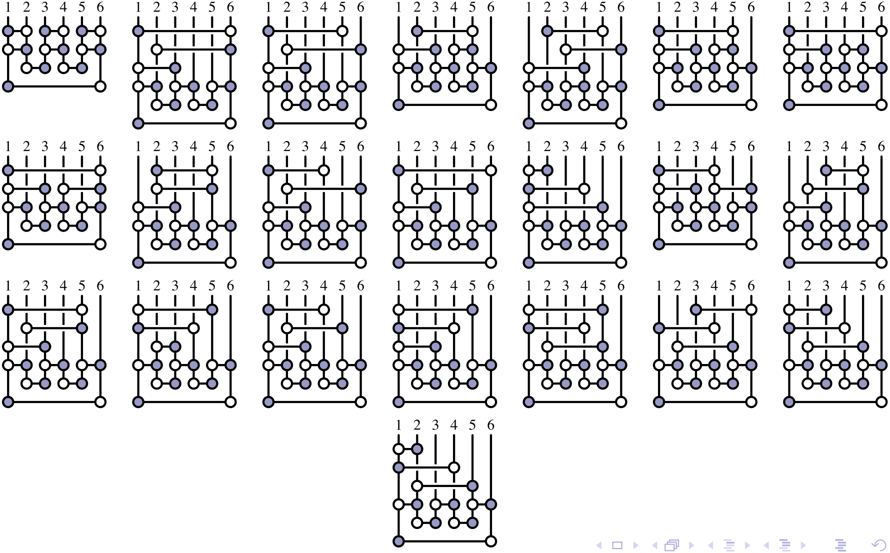

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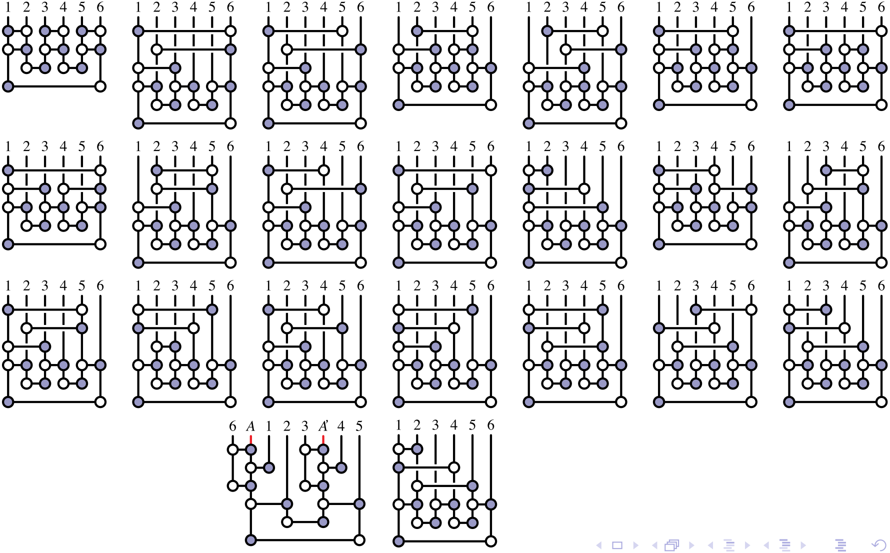

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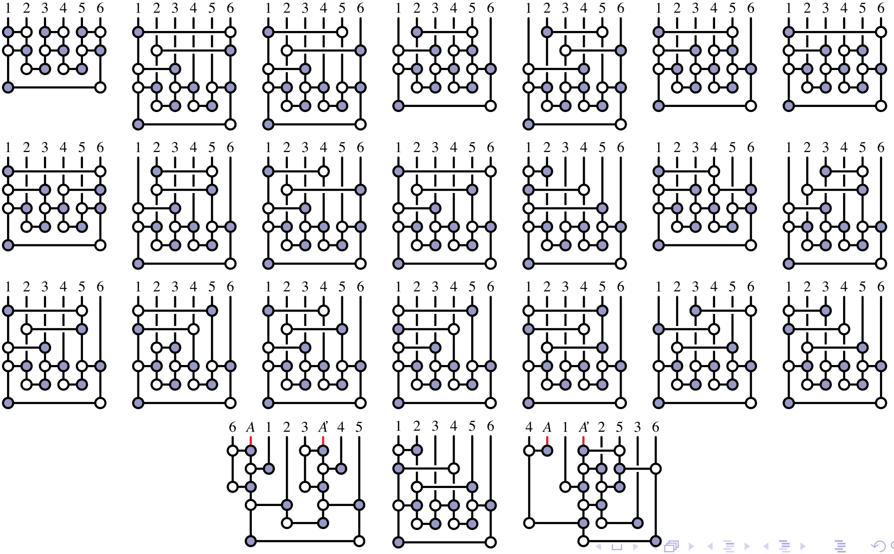

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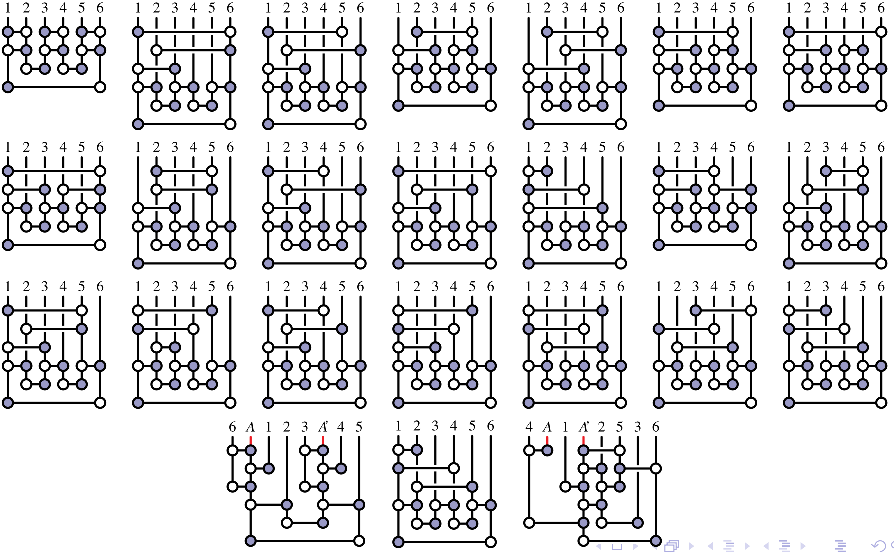

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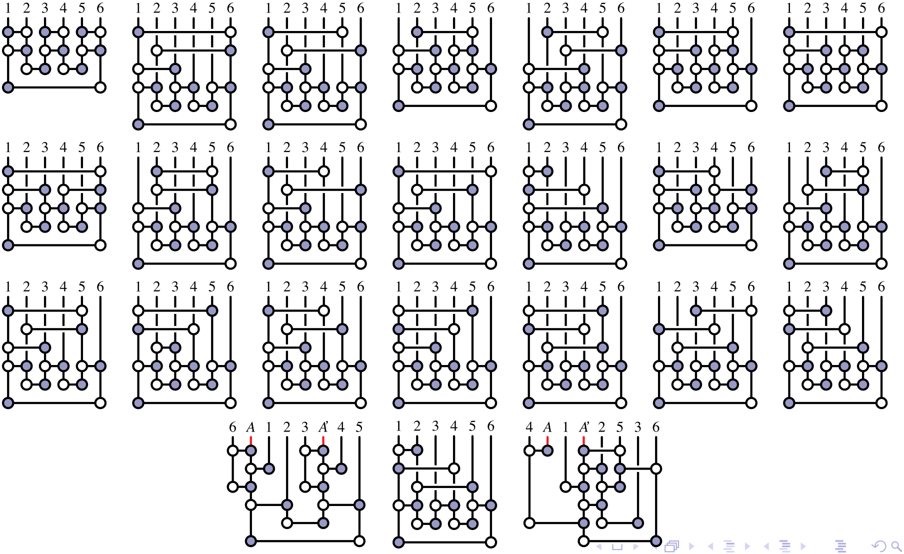

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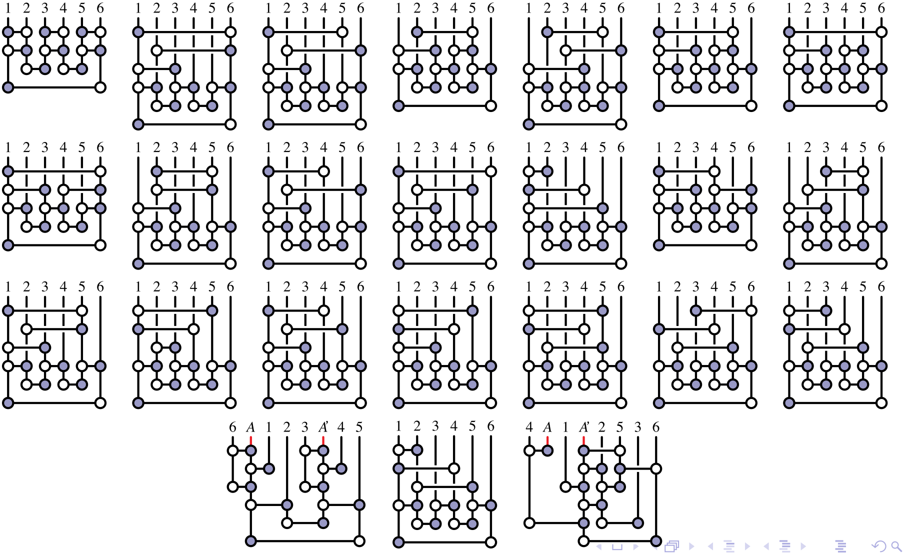

Enumeration of All (ten) 'Leading Singularities' of $G(3,6)$

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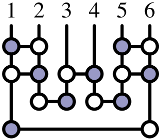
$$f_1 \equiv \oint_{(123)=0} \Omega_1 = \left.\frac{\delta^{3\times 4}(C^*\!\cdot\!\widetilde{\eta})\,\delta^{2\times 2}(\lambda\!\cdot\!\widetilde{\lambda})}{(234)(345)(456)(561)(612)}\right|_{C^*} \qquad C^* \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & [56] & [64] & [45] \end{pmatrix}$$

$$= \frac{\delta^{3\times 4}(C^*\!\cdot\!\widetilde{\eta})\,\delta^{2\times 2}(\lambda\!\cdot\!\widetilde{\lambda})}{\langle 23\rangle\,[56]\,\langle 3|4{+}5|6]s_{456}\langle 1|5{+}6|4]\langle 12\rangle\,[45]}$$



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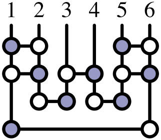

$$f_2\equiv\oint_{(123)=0}\Omega_2=\left.\frac{(235)\,\delta^{3\times4}(C^*\!\cdot\widetilde{\eta})\,\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{(136)(156)(234)(245)(256)(345)}\right|_{C^*}\qquad C^*\equiv\begin{pmatrix}\lambda_1^1&\lambda_2^1&\lambda_3^1&\lambda_4^1&\lambda_5^1&\lambda_6^1\\\lambda_1^2&\lambda_2^2&\lambda_3^2&\lambda_4^2&\lambda_5^2&\lambda_6^2\\0&0&0&[56]&[64]&[45]\end{pmatrix}$$

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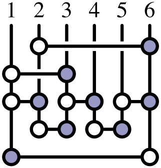

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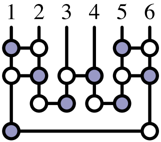

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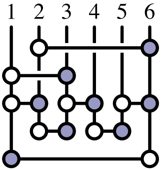
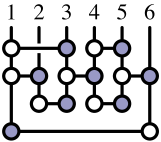
$$= \frac{\langle23\rangle\,[64]\;\delta^{3\times4}(C^*\!\cdot\!\widetilde{\eta})\,\delta^{2\times2}(\lambda\!\cdot\!\widetilde{\lambda})}{\langle13\rangle\,[45]\,\langle1|5{+}6|4]\langle23\rangle\,[56]\,\langle2|4{+}5|6]\langle2|5{+}6|4]\langle3|4{+}5|6]}$$



$$f_3 \equiv \oint_{(123)=0} \Omega_4 = \frac{(145)\;\delta^{3\times4}(C^*\!\cdot\!\widetilde{\eta})\,\delta^{2\times2}(\lambda\!\cdot\!\widetilde{\lambda})}{(124)(136)(156)(245)(345)(456)}\Bigg|_{C^*} \qquad C^* \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & [56] & [64] & [45] \end{pmatrix}$$

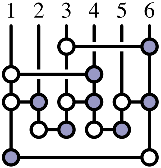
$$= \frac{\langle1|4{+}5|6]\;\delta^{3\times4}(C^*\!\cdot\!\widetilde{\eta})\,\delta^{2\times2}(\lambda\!\cdot\!\widetilde{\lambda})}{\langle12\rangle\,[56]\,\langle13\rangle\,[45]\,\langle1|5{+}6|4]\langle2|4{+}5|6]\langle3|4{+}5|6]s_{456}}$$



Enumeration of All (ten) 'Leading Singularities' of $G(3,6)$

$$f_4 \equiv \oint\limits_{(123)=0} \Omega_5 = \left.\frac{(135)\,\delta^{3\times4}\big(C^{*}\!\cdot\!\widetilde{\eta}\big)\delta^{2\times2}\big(\lambda\!\cdot\!\widetilde{\lambda}\big)}{(124)(145)(156)(236)(345)(356)}\right|_{C^{*}} \qquad C^{*} \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & [56] & [64] & [45] \end{pmatrix}$$

$$= \frac{\langle 13\rangle\,[64]\;\delta^{3\times4}\big(C^{*}\!\cdot\!\widetilde{\eta}\big)\delta^{2\times2}\big(\lambda\!\cdot\!\widetilde{\lambda}\big)}{\langle 12\rangle\,[56]\,\langle 1|4{+}5|6]\langle 1|5{+}6|4]\langle 23\rangle\,[45]\,\langle 3|4{+}5|6]\langle 3|5{+}6|4]}$$



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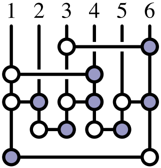
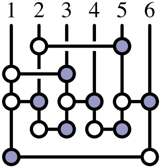
$$f_4 \equiv \oint_{(123)=0} \Omega_5 = \left.\frac{(135)\,\delta^{3\times4}(C^*\!\cdot\widetilde{\eta})\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{(124)(145)(156)(236)(345)(356)}\right|_{C^*} \qquad C^* \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & [56] & [64] & [45] \end{pmatrix}$$

$$= \frac{\langle 13\rangle\,[64]\;\delta^{3\times4}(C^*\!\cdot\widetilde{\eta})\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{\langle 12\rangle\,[56]\,\langle 1|4{+}5|6]\langle 1|5{+}6|4]\langle 23\rangle\,[45]\,\langle 3|4{+}5|6]\langle 3|5{+}6|4]}$$



$$f_5 \equiv \oint_{(123)=0} \Omega_9 = \left.\frac{(125)\,\delta^{3\times4}(C^*\!\cdot\widetilde{\eta})\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{(134)(156)(245)(256)(16(25)\bigcap(34))}\right|_{C^*} \qquad C^* \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & [56] & [64] & [45] \end{pmatrix}$$

$$= \frac{\langle 12\rangle\,[64]\;\delta^{3\times4}(C^*\!\cdot\widetilde{\eta})\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{\langle 13\rangle\,[56]\,\langle 1|5{+}6|4]\langle 2|4{+}5|6]\langle 2|5{+}6|4]\big(\langle 23\rangle\,[56]\,\langle 1|5{+}6|4]-\langle 12\rangle\,[45]\,\langle 3|4{+}5|6]\big)}$$



Enumeration of All (ten) 'Leading Singularities' of $G(3,6)$

$$f_4 \equiv \oint_{(123)=0} \Omega_5 = \left.\frac{(135)\,\delta^{3\times4}(C^{*}\!\cdot\widetilde{\eta})\,\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{(124)(145)(156)(236)(345)(356)}\right|_{C^{*}} \qquad C^{*} \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & [56] & [64] & [45] \end{pmatrix}$$

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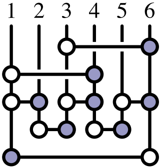
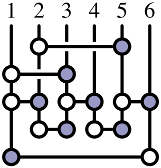


$$f_5 \equiv \oint_{(123)=0} \Omega_9 = \left.\frac{(125)\,\delta^{3\times4}(C^{*}\!\cdot\widetilde{\eta})\,\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{(134)(156)(245)(256)(16(25)\bigcap(34))}\right|_{C^{*}} \qquad C^{*} \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & [56] & [64] & [45] \end{pmatrix}$$

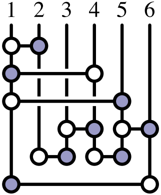
$$= \frac{\langle 12\rangle\,[64]\;\delta^{3\times4}(C^{*}\!\cdot\widetilde{\eta})\,\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{\langle 13\rangle\,[56]\,\langle 1|5{+}6|4]\langle 2|4{+}5|6]\langle 2|5{+}6|4]\big(\langle 23\rangle\,[56]\,\langle 1|5{+}6|4]-\langle 12\rangle\,[45]\,\langle 3|4{+}5|6]\big)}$$



$$C^{*} \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & [56] & [64] & [45] \end{pmatrix}$$

$$f_6 \equiv \oint_{(123)=0} \Omega_{12} = \left.\frac{(134)^2(456)\,\delta^{3\times4}(C^{*}\!\cdot\widetilde{\eta})\,\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{(124)(145)(146)(156)(234)(345)(346)(356)}\right|_{C^{*}}$$

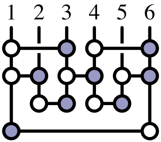
$$= \frac{\langle 13\rangle^2 s_{456}\,\delta^{3\times4}(C^{*}\!\cdot\widetilde{\eta})\,\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{\langle 12\rangle\langle 1|4{+}5|6]\langle 1|4{+}6|5]\langle 1|5{+}6|4]\langle 23\rangle\langle 3|4{+}5|6]\langle 3|4{+}6|5]\langle 3|5{+}6|4]}$$



Enumeration of All (ten) 'Leading Singularities' of $G(3,6)$

$$f_7 \equiv \oint\limits_{(123)=0} \Omega_{13} = \frac{(145)^2\,\delta^{3\times4}\big(C^*\!\cdot\widetilde{\eta}\big)\delta^{2\times2}\big(\lambda\cdot\widetilde{\lambda}\big)}{(125)(134)(146)(156)(245)(345)(456)}\Bigg|_{C^*} \qquad C^* \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & [56] & [64] & [45] \end{pmatrix}$$

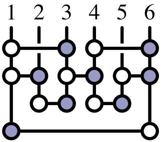
$$= \frac{\langle 1|4{+}5|6]^2\,\delta^{3\times4}\big(C^*\!\cdot\widetilde{\eta}\big)\delta^{2\times2}\big(\lambda\cdot\widetilde{\lambda}\big)}{\langle 12\rangle\,[64]\,\langle 13\rangle\,[56]\,\langle 1|4{+}6|5]\langle 1|5{+}6|4]\langle 2|4{+}5|6]\langle 3|4{+}5|6]s_{456}}$$



Enumeration of All (ten) 'Leading Singularities' of $G(3,6)$

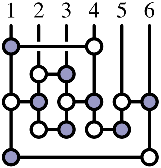
$$f_7 \equiv \oint_{(123)=0} \Omega_{13} = \left.\frac{(145)^2\,\delta^{3\times4}(C^*\cdot\widetilde{\eta})\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{(125)(134)(146)(156)(245)(345)(456)}\right|_{C^*} \qquad C^* \equiv \begin{pmatrix} \lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2 \\ 0 & 0 & 0 & [56] & [64] & [45] \end{pmatrix}$$

$$= \frac{\langle 1|4{+}5|6]^2\,\delta^{3\times4}(C^*\cdot\widetilde{\eta})\delta^{2\times2}(\lambda\cdot\widetilde{\lambda})}{\langle 12\rangle\,[64]\,\langle 13\rangle\,[56]\,\langle 1|4{+}6|5]\langle 1|5{+}6|4]\langle 2|4{+}5|6]\langle 3|4{+}5|6]s_{456}}$$



$$f_8 \equiv \oint_{(14(23)\cap(56))=0} \Omega_{16} = \int \frac{d\alpha_1}{\alpha_1}\wedge\cdots\wedge\frac{d\alpha_8}{\alpha_8}\;\delta^{3\times4}(C(\alpha)\cdot\widetilde{\eta})\delta^{3\times2}(C(\alpha)\cdot\widetilde{\lambda})\delta^{2\times3}(\lambda\cdot C^{\perp}(\alpha))$$

$$C(\alpha) \equiv \begin{pmatrix} 1 & \alpha_6 & \alpha_6\,\alpha_7 & 0 & 0 & \alpha_1 \\ 0 & 1 & \alpha_5{+}\alpha_7 & 0 & \alpha_2 & \alpha_2\,\alpha_4 \\ \alpha_8 & 0 & 0 & 1 & \alpha_3 & \alpha_3\,\alpha_4 \end{pmatrix}$$



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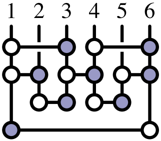
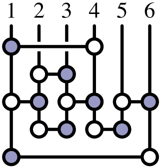
$$f_7 \equiv \oint\limits_{(123)=0} \Omega_{13} = \left.\frac{(145)^2\,\delta^{3\times4}\big(C^*\!\cdot\widetilde{\eta}\big)\delta^{2\times2}\big(\lambda\cdot\widetilde{\lambda}\big)}{(125)(134)(146)(156)(245)(345)(456)}\right|_{C^*} \qquad C^*\equiv\begin{pmatrix}\lambda_1^1 & \lambda_2^1 & \lambda_3^1 & \lambda_4^1 & \lambda_5^1 & \lambda_6^1\\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \lambda_5^2 & \lambda_6^2\\ 0 & 0 & 0 & [56] & [64] & [45]\end{pmatrix}$$

$$= \frac{\langle 1|4{+}5|6]^2\,\delta^{3\times4}\big(C^*\!\cdot\widetilde{\eta}\big)\delta^{2\times2}\big(\lambda\cdot\widetilde{\lambda}\big)}{\langle 12\rangle\,[64]\,\langle 13\rangle\,[56]\,\langle 1|4{+}6|5]\langle 1|5{+}6|4]\langle 2|4{+}5|6]\langle 3|4{+}5|6]s_{456}}$$



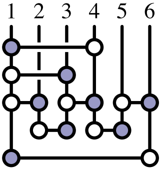
$$f_8 \equiv \oint\limits_{(14(23)\cap(56))=0} \Omega_{16} = \int \frac{d\alpha_1}{\alpha_1}\wedge\cdots\wedge\frac{d\alpha_8}{\alpha_8}\;\delta^{3\times4}\big(C(\alpha)\cdot\widetilde{\eta}\big)\delta^{3\times2}\big(C(\alpha)\cdot\widetilde{\lambda}\big)\delta^{2\times3}\big(\lambda\cdot C^{\perp}(\alpha)\big)$$

$$C(\alpha)\equiv\begin{pmatrix}1 & \alpha_6 & \alpha_6\,\alpha_7 & 0 & 0 & \alpha_1\\ 0 & 1 & \alpha_5{+}\alpha_7 & 0 & \alpha_2 & \alpha_2\,\alpha_4\\ \alpha_8 & 0 & 0 & 1 & \alpha_3 & \alpha_3\,\alpha_4\end{pmatrix}$$



$$f_9 \equiv \oint\limits_{(14(23)\cap(56))=0} \Omega_{18} = \int \frac{d\alpha_1}{\alpha_1}\wedge\cdots\wedge\frac{d\alpha_8}{\alpha_8}\;\delta^{3\times4}\big(C(\alpha)\cdot\widetilde{\eta}\big)\delta^{3\times2}\big(C(\alpha)\cdot\widetilde{\lambda}\big)\delta^{2\times3}\big(\lambda\cdot C^{\perp}(\alpha)\big)$$

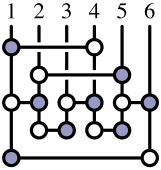
$$C(\alpha)\equiv\begin{pmatrix}1 & \alpha_5 & \alpha_7 & 0 & 0 & \alpha_1\\ 0 & 1 & \alpha_4 & 0 & \alpha_2 & \alpha_2\,\alpha_6\\ \alpha_8 & 0 & 0 & 1 & \alpha_3 & \alpha_3\,\alpha_6\end{pmatrix}$$



Enumeration of All (ten) ‘Leading Singularities’ of $G(3,6)$

$$f_{10} \equiv \oint\limits_{z=0} \Omega_{20} = \int \frac{d\alpha_1}{\alpha_1} \wedge \cdots \wedge \frac{d\alpha_8}{\alpha_8}\; \delta^{3\times4}\big(C(\alpha)\cdot\widetilde{\eta}\big)\delta^{3\times2}\big(C(\alpha)\cdot\widetilde{\lambda}\big)\delta^{2\times3}\big(\lambda\cdot C^{\perp}(\alpha)\big)$$

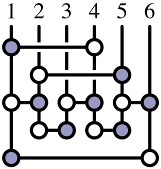
$$C(\alpha) \equiv \begin{pmatrix} \alpha_6\,\alpha_8 & \alpha_1 & 1 & \alpha_6 & \alpha_1\,\alpha_7 & 0 \\ \alpha_8 & 0 & 0 & 1 & \alpha_5 & \alpha_4 \\ \alpha_3 & \alpha_2 & 0 & 0 & \alpha_2\,\alpha_7 & 1 \end{pmatrix}$$



Enumeration of All (ten) ‘Leading Singularities’ of $G(3,6)$

$$f_{10} \equiv \oint_{z=0} \Omega_{20} = \int \frac{d\alpha_1}{\alpha_1} \wedge \cdots \wedge \frac{d\alpha_8}{\alpha_8}\; \delta^{3\times 4}\big(C(\alpha)\cdot\widetilde{\eta}\big)\delta^{3\times 2}\big(C(\alpha)\cdot\widetilde{\lambda}\big)\delta^{2\times 3}\big(\lambda\cdot C^{\perp}(\alpha)\big)$$

$$C(\alpha) \equiv \begin{pmatrix} \alpha_6\,\alpha_8 & \alpha_1 & 1 & \alpha_6 & \alpha_1\,\alpha_7 & 0 \\ \alpha_8 & 0 & 0 & 1 & \alpha_5 & \alpha_4 \\ \alpha_3 & \alpha_2 & 0 & 0 & \alpha_2\,\alpha_7 & 1 \end{pmatrix}$$



On-Shell Physics/Grassmannian Geometry Correspondence

$$f_\Gamma \equiv \prod_i \Big(\sum_{h_i, q_i} \int d^3\mathrm{LIPS}_i \Big) \prod_v \mathcal{A}_v \equiv \int \Omega_C \; \delta(C, p, h)$$

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On-Shell Physics

- on-shell diagrams
- physical symmetries
  - trivial symmetries (identities)

$\Leftrightarrow$

Grassmannian Geometry

- $\{$strata $C \in G(k,n)$, volume-form $\Omega_C\}$
- volume-preserving diffeomorphisms
  - cluster coordinate mutations

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Important Open Questions (for math *and* physics)

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Important Open Questions (for math *and* physics)

- how many functions exist?

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- how many functions exist? (how to name them?)

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- what (functional) relations do they satisfy?

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Important Open Questions (for math *and* physics)

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- what are their (infinite-dimensional) symmetries?

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- what (functional) relations do they satisfy?
- what are their (infinite-dimensional) symmetries?
  - do these extend to entire *amplitudes*?

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$\Leftrightarrow$

Grassmannian Geometry

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- volume-preserving diffeomorphisms
  - cluster coordinate mutations

Important Open Questions (for math *and* physics)

- how many functions exist? (how to name them?)
- what (functional) relations do they satisfy?
- what are their (infinite-dimensional) symmetries?
  - do these extend to entire *amplitudes*?
- do loop-level recursion relations exist?

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$\Leftrightarrow$

Grassmannian Geometry

- $\{$strata $C \in G(k,n)$, volume-form $\Omega_C\}$
- volume-preserving diffeomorphisms
  - cluster coordinate mutations

Important Open Questions (for math *and* physics)

- how many functions exist? (how to name them?)
- what (functional) relations do they satisfy?
- what are their (infinite-dimensional) symmetries?
  - do these extend to entire *amplitudes*?
- do loop-level recursion relations exist?

On-Shell Physics/Grassmannian Geometry Correspondence

$$f_\Gamma \equiv \prod_i \Big(\sum_{h_i, q_i} \int d^3\mathrm{LIPS}_i \Big) \prod_v \mathcal{A}_v \equiv \int \Omega_C \; \delta(C, p, h)$$

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- on-shell diagrams
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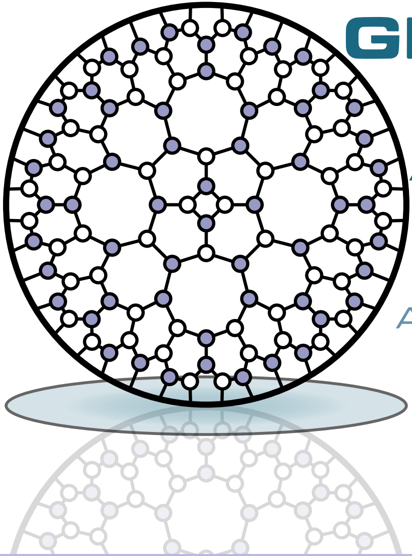

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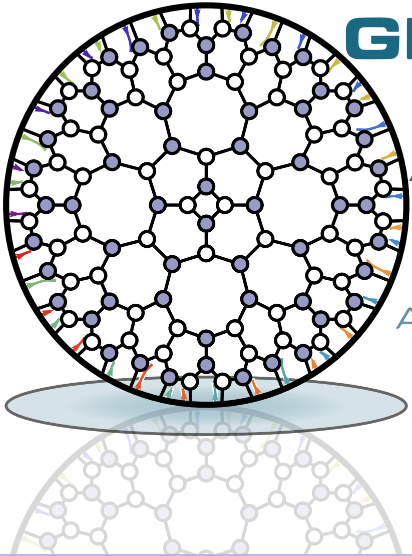

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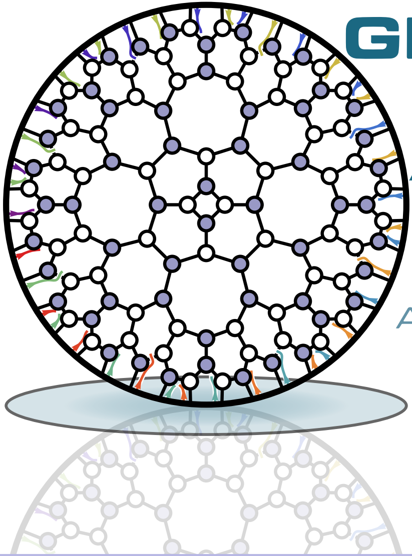

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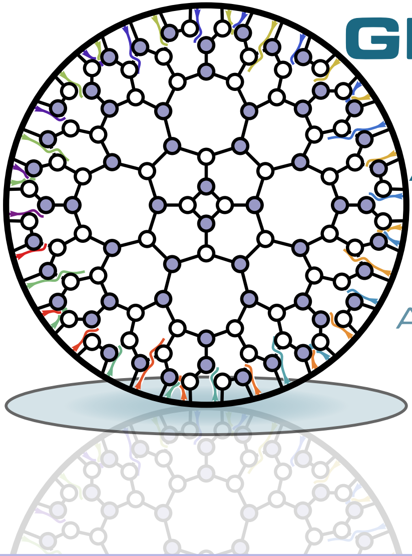

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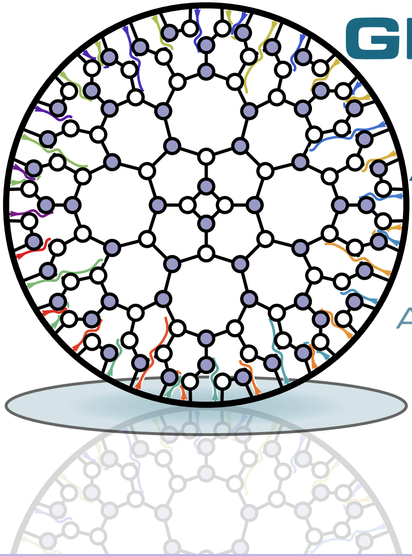

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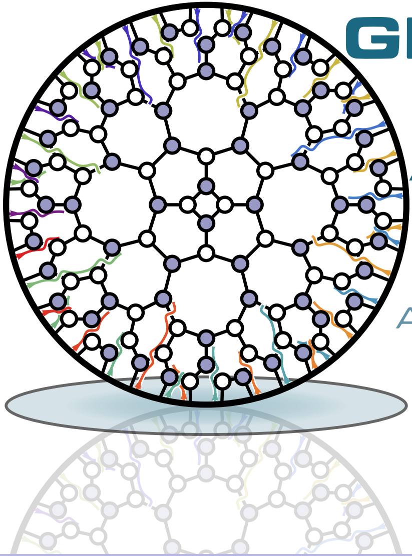

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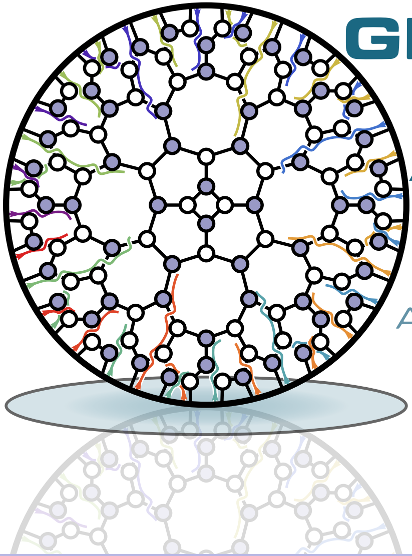

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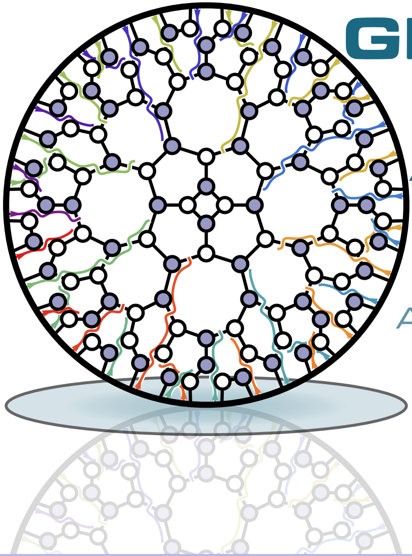

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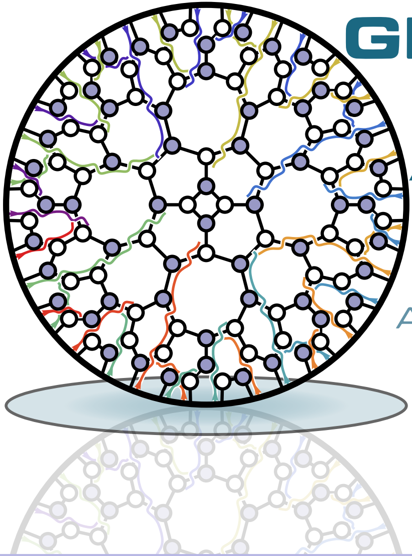

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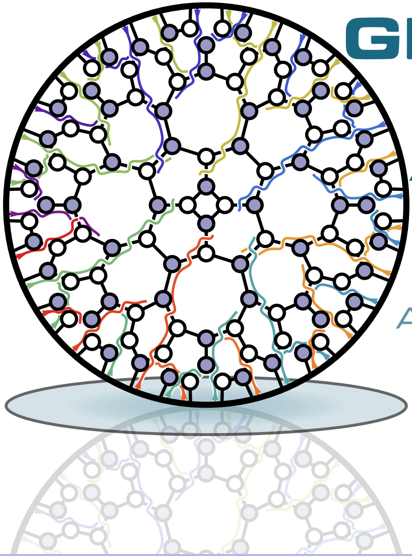

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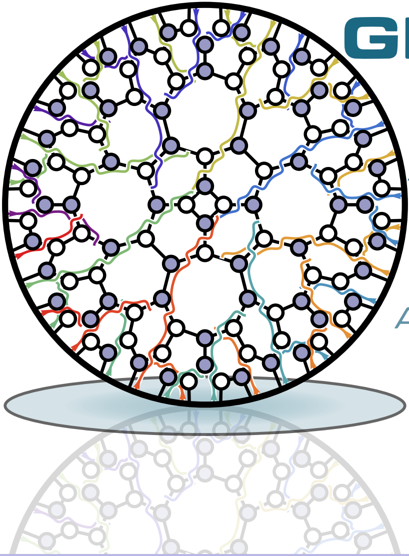

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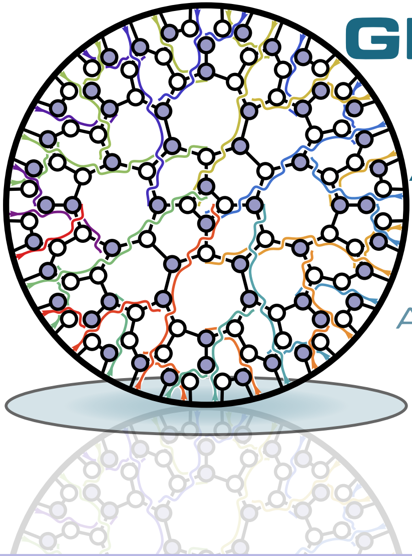

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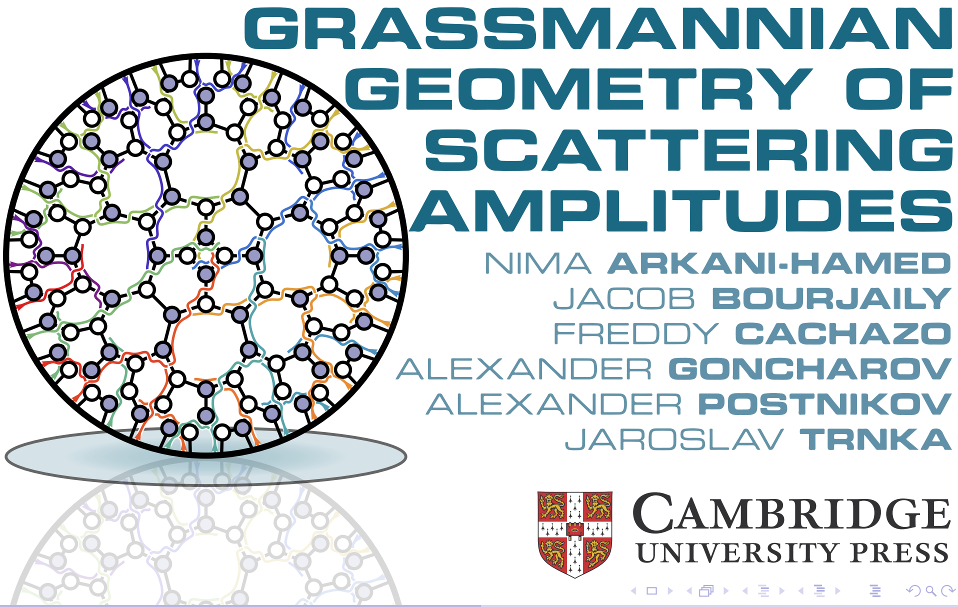
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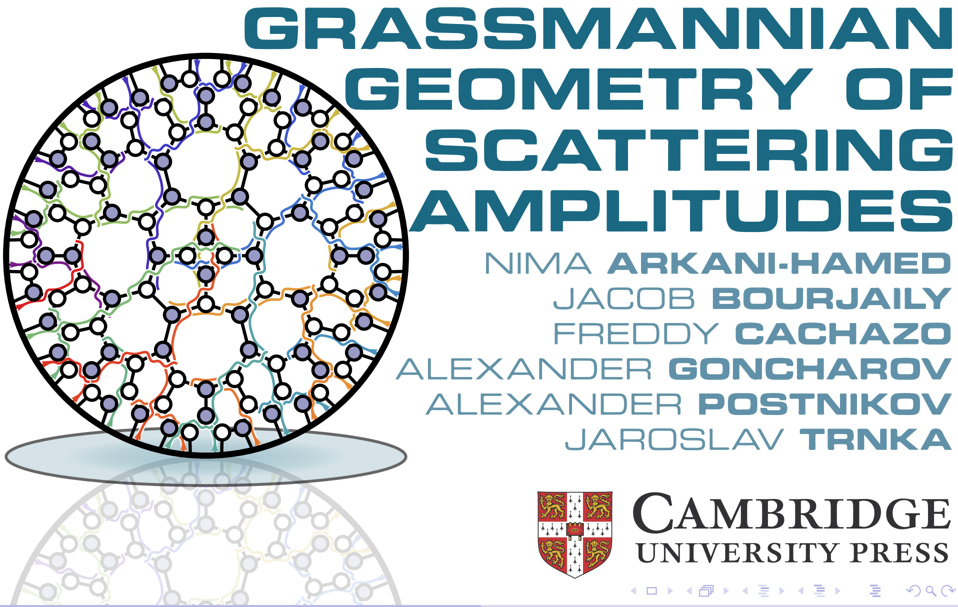
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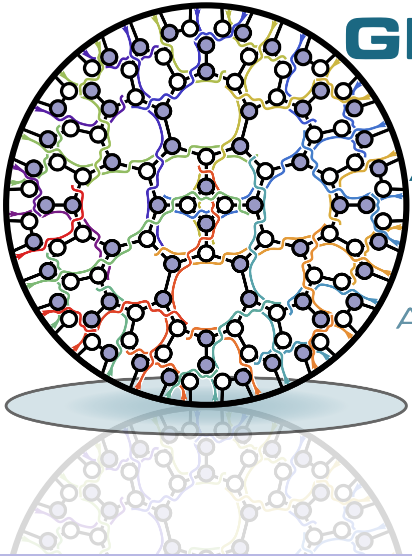

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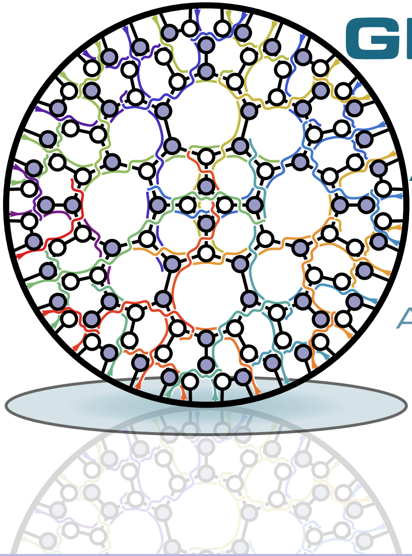

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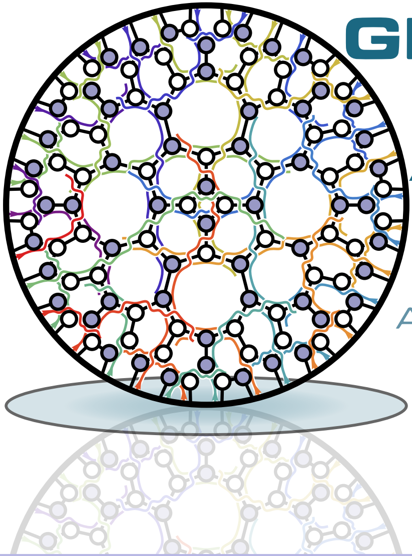

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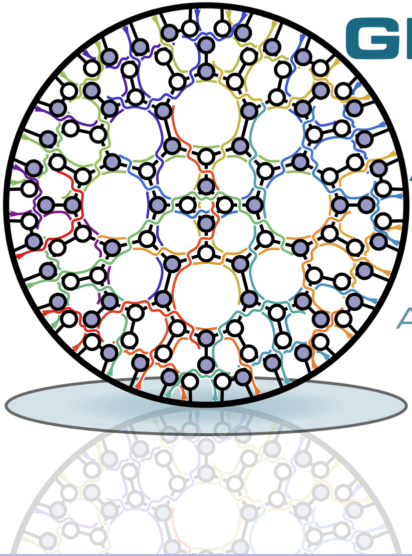

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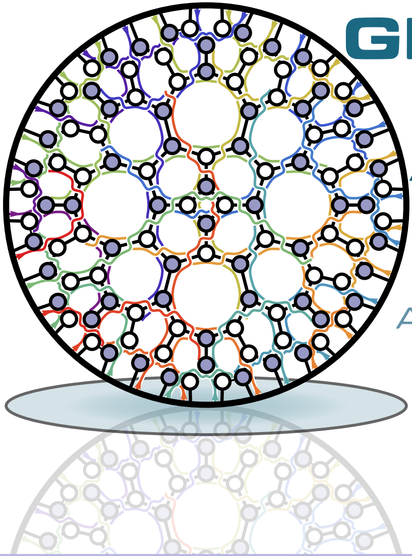

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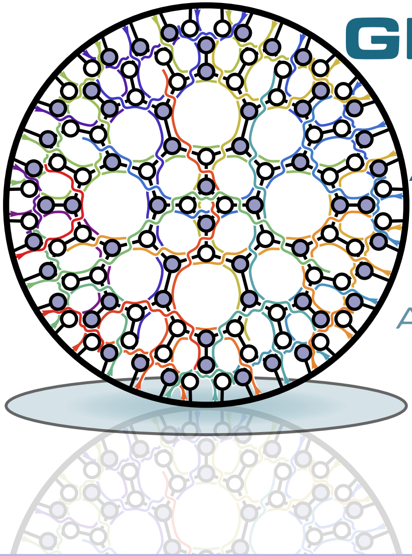

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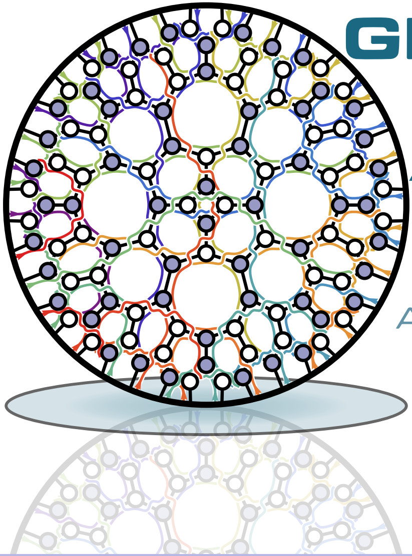

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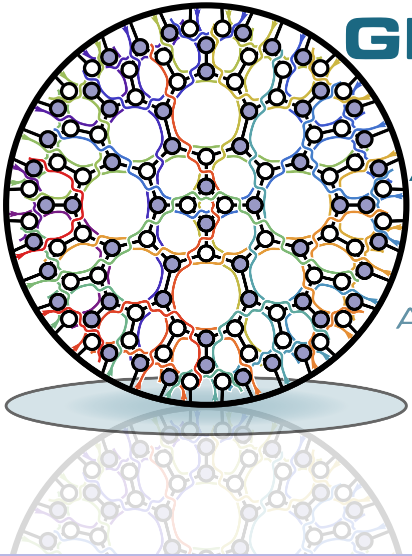

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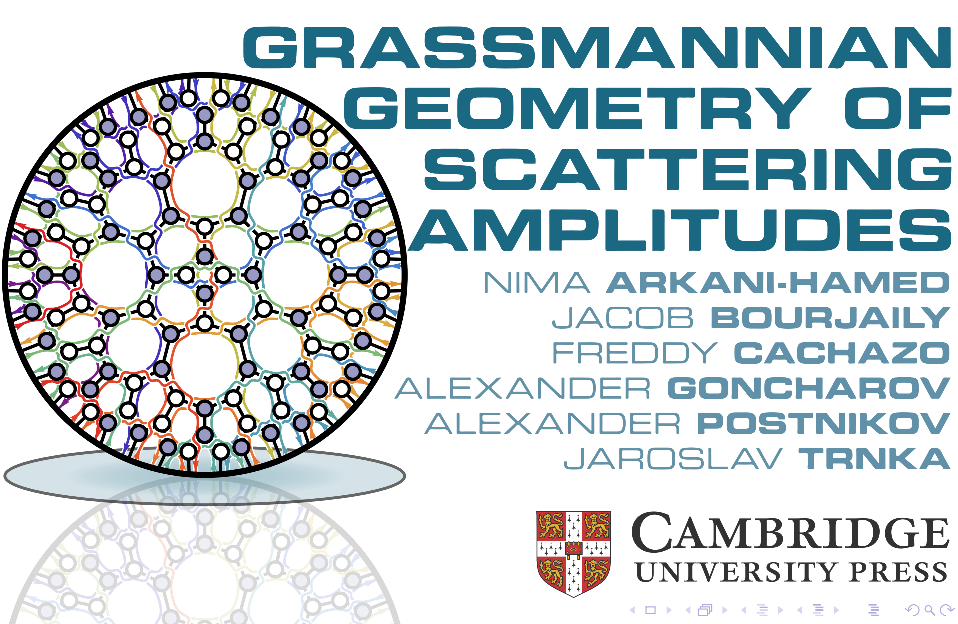
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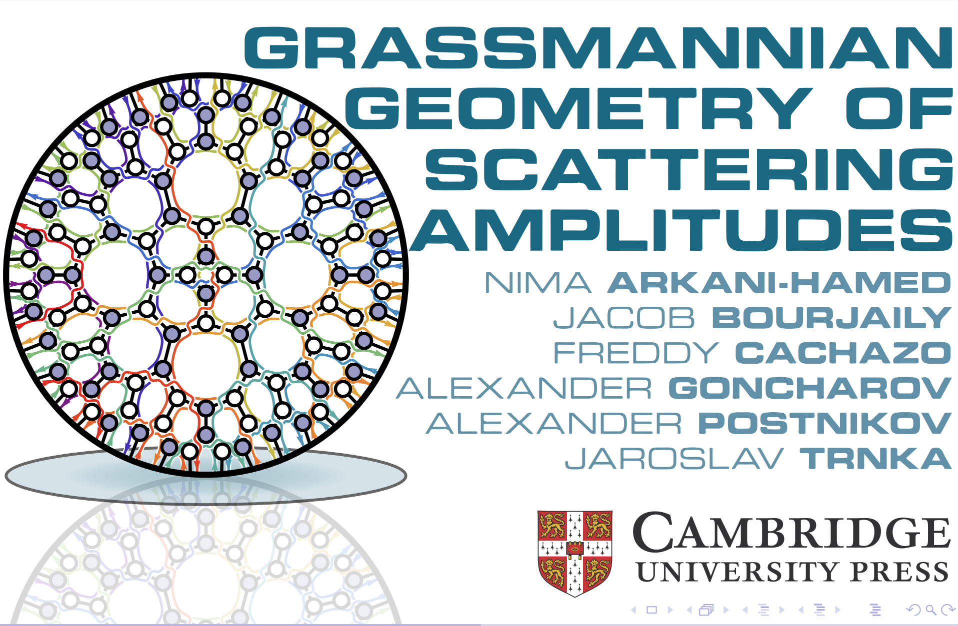
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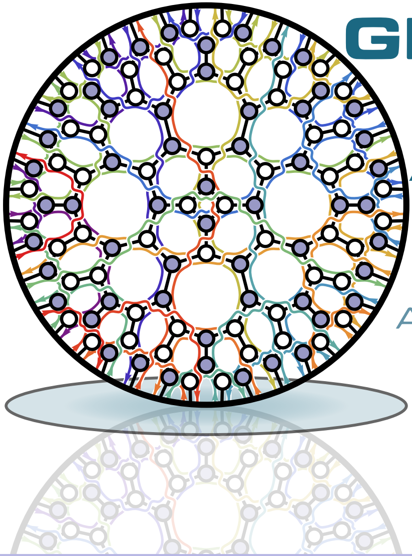

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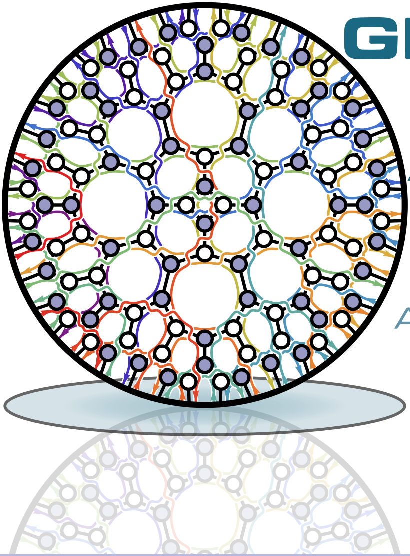

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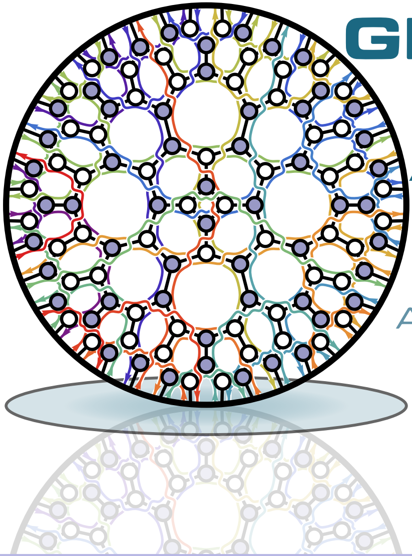

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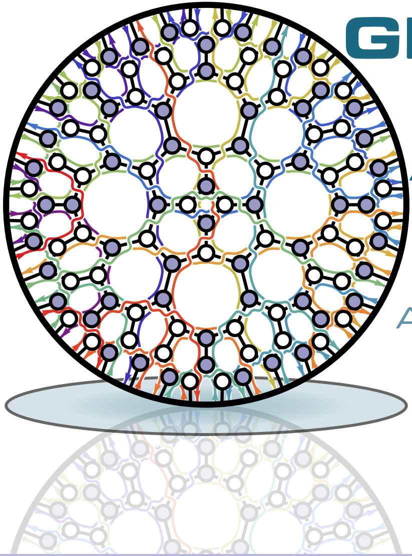

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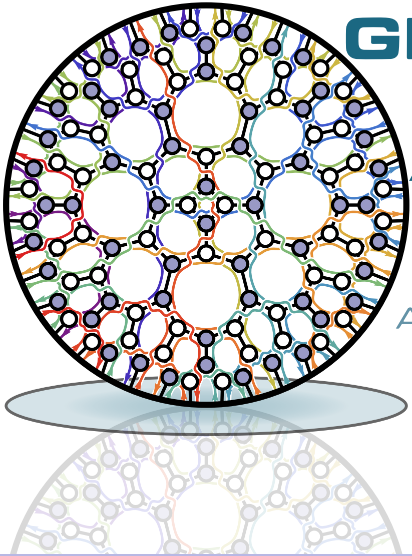

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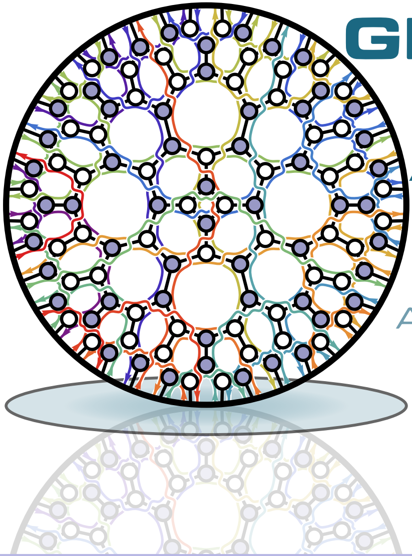

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