Hyperbolic Geometry and Scattering Amplitudes

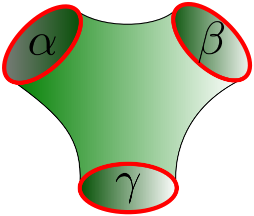
Giulio Salvatori
with Sergio Cacciatori, arXiv:1803.05809

Amplitudes 2018 Summer School

UNIVERSITÀ DEGLI STUDI DI MILANO
FACOLTÀ DI SCIENZE E TECNOLOGIE

INFN
Istituto Nazionale di Fisica Nucleare

Hyperbolic approach to the Moduli problem



$$\alpha, \beta, \gamma \geq 0$$

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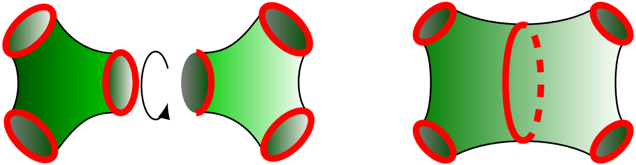

Fenchel-Nielsen coordinates ($\alpha \geq 0, 0 \leq \theta \leq 2\pi$)

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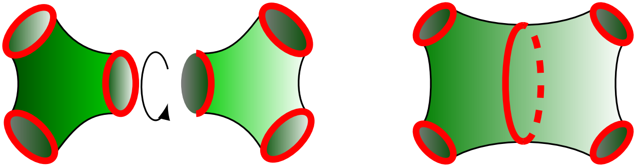

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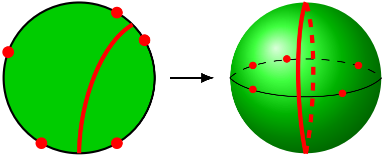
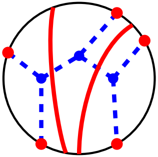
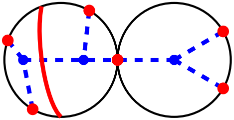


Schottky double: A pants decomposition of the double gives arcs on the bordered surface.

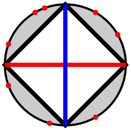
The Associahedron... (arXiv:1711.09102)

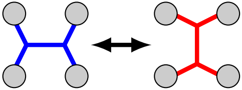


Non intersecting arcs $\Leftrightarrow$
Propagators of a Feynman diagram
Vertices of the Associahedron $\Leftrightarrow$
Planar tree Feynman diagrams



$$\partial \mathcal{A}_n \sim \mathcal{A}_{n_L} \times \mathcal{A}_{n_R}$$





$$\Omega(\mathcal{A}_n) = \sum_g \operatorname{sgn}(g) \bigwedge_{I \in g} \frac{d\ S_I}{S_I}$$

$$= m_n\ d^{n-3} S$$

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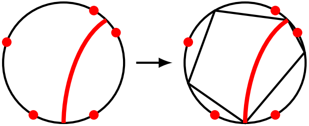

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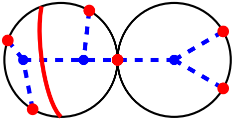

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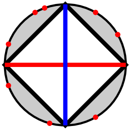

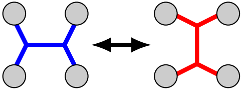

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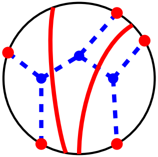

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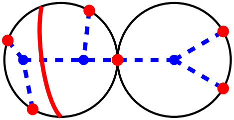

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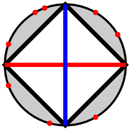

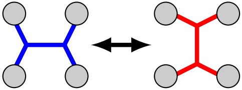

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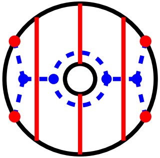
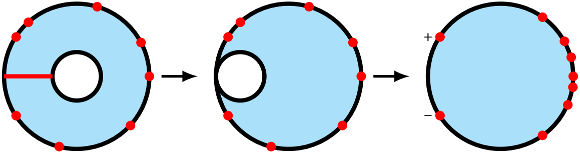
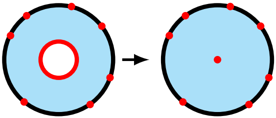


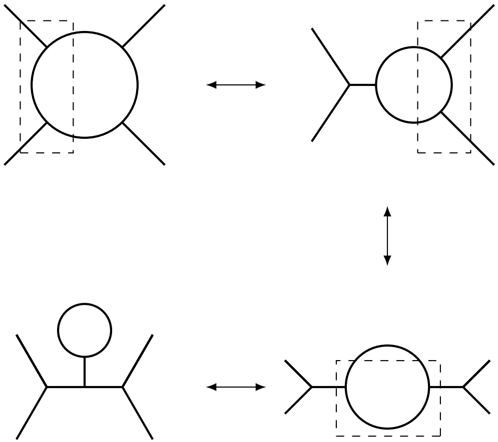
Non intersecting arcs ⇔
Propagators of 1-loop planars diagrams
Vertices of the Halohedron ⇔
1-loop planar diagrams



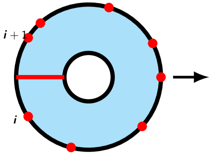


Cut Facets ⇔ Forward limit
“UV” facets have no immediate physical interpretation

Mutations at 1-loop

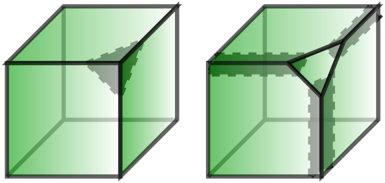


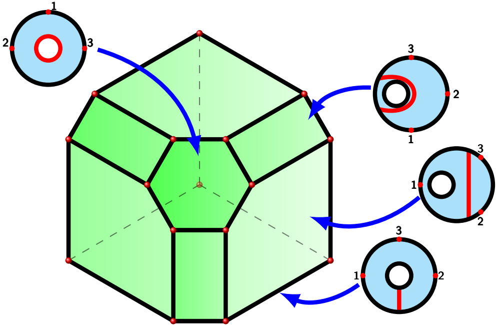
Halohedron in Abstract Space



X_i.

To any other arc I is associated a function X_I. The region $X_I > 0$ is an Halohedron (Devadoss, 2010).





1-loop integrand from the Halohedron

$$\Omega(H_n) = \sum_g \mathrm{sgn}(g) \bigwedge_{I\in g} \frac{d\ X_I}{X_I} = d^n X \sum_g \prod_{I\in g} \frac{1}{X_I}$$

The sum is over all 1-loop planar diagrams: tadpoles, internal and external bubbles...we can kill them sending the corresponding variables X_I to infinity.

- $X_T, X_{UV}, X_{\text{ext bubble}} \to \infty$
- $X_I \to S_I$
- $\Omega(H_n) \to \mathcal{I}_n d^n X$

Triangulations produce new formulae, e.g.

$$\mathcal{I}_4 = -\frac{1}{\ell_1^2\ell_2^2\ell_3^2\ell_4^2} + \left(\frac{(\ell_1^2 + S_{12})(\ell_3^2 + S_{12})}{S_{12}^2\ell_2^2\ell_3^2\ell_4^2(\ell_3^2 - \ell_1^2)} + cyclical\right)$$

Integrand with double poles from a dlog!

Future Directions

- Higher loop order: Moduli spaces are not polytopes anymore, hide problem "at infinity"?
- Partial amplitudes with two colour orderings: Intersection numbers, Intersecting Halohedra? (Mizera, 2017 and He, 2018)
- Different triangulations: Forward limit recursions? (He, 2015)
- Scattering Equations: Pushforward interpretation of CHY 1-loop formulae?

Thanks for your attention!