Holographic Entanglement

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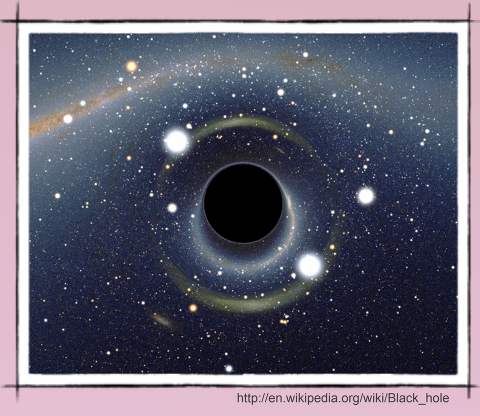
Holography

- Originated with black holes:
  Amount of information stored in a black hole is given by its surface area, rather than its volume
- Holographic principle:
  In a theory of gravity, the amount of information needed to describe the physics in any region is bounded by the surface area of that region
- Best example:



http://en.wikipedia.org/wiki/Black_hole

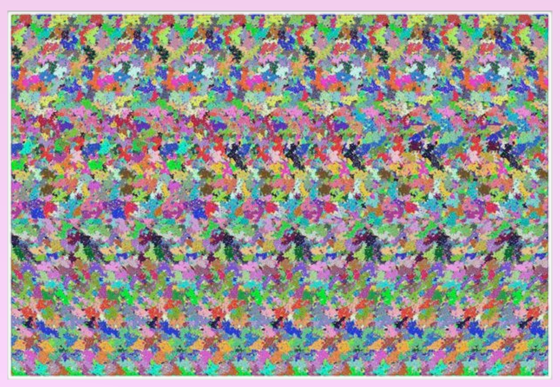
The AdS/CFT duality [Maldacena, 1997]

String theory (which contains gravity) in Anti de Sitter (AdS) spacetime is equivalent to a quantum field theory (CFT) on the boundary of AdS.

Soup can analogy:
quantum field theory (CFT) = can label;
string theory (in AdS) = soup inside can;
but unlike ordinary soup can, where label just gives a rough idea, here 'label is everything'

This duality is called "holographic" because a lower-dimensional theory encodes a higher-dimensional one.

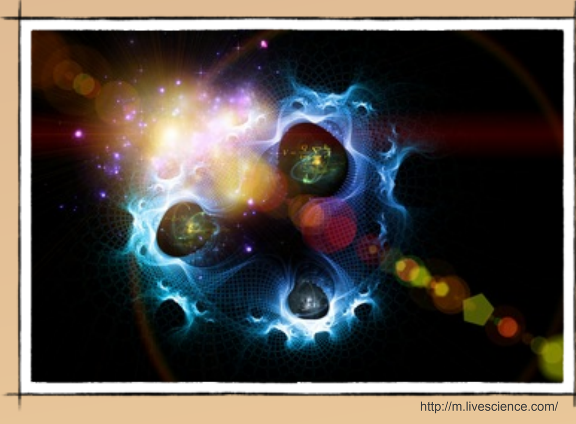
Better analogy: stereogram
(3D image emerges from 2D correlations)
but AdS/CFT is infinitely more complicated



- One of the most profound advances in theoretical physics in the last 50 years
- Invaluable for learning about strongly coupled field theories as well as about quantum gravity
- To realise the full potential requires good understanding of the "dictionary" between the two sides (e.g. how does spacetime emerge from CFT).

Entanglement

- The most quintessentially quantum feature of quantum mechanics (absent in classical physics)
- Present day applications:
  - quantum cryptography
  - quantum teleportation
  - quantum optics
  - condensed-matter nano-physics
  - quantum dense coding
  - and many more...
- Hence a crucial quantity for understanding quantum matter, quantum information, quantum computation, and perhaps even quantum gravity!



http://m.livescience.com/

- Definition:
  A state is entangled if:
  - it is not mixture of product states, or
  - it is a resource for non-classical tasks
- For such states, the best possible knowledge of a whole does not include best possible knowledge of its parts!
- Classic example: "EPR pair":
  measuring spin of one component instantly determines the spin of the other, even arbitrarily far away.

http://solhub.org/itschool.php

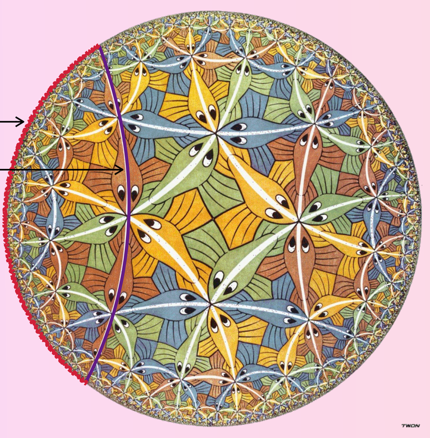
- Amount of entanglement characterised by Entanglement Entropy
- This is very difficult to calculate directly, but Holography comes to the rescue...

Covariant Holographic Entanglement Entropy

Ryu-Takayanagi prescription

- Applies only to static configurations
  => consider a single snapshot in time
- Entanglement entropy of a region A on the boundary is given by the area of a minimal surface M in the bulk of AdS:

e.g. AdS space:
boundary region A
minimal surface M



Spatial geometry of AdS is nicely captured by this Escher drawing: each fish has the same proper size and any point can be thought of as the center.

The minimal surface is analogous to a soap bubble, since it is the surface with smallest area for the given boundary conditions.

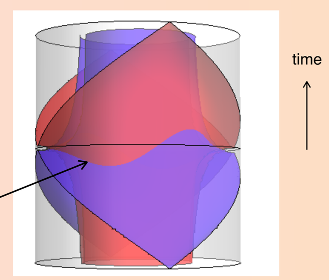
- This intriguing connection has many useful applications to our understanding of both sides.
- However, it has one flaw: it does not work in general time-dependent setting.

Hubeny-Rangamani-Takayanagi prescription

- Generalizes Ryu-Takayanagi, using general covariance as a guiding principle.
- Applies to arbitrary time-dependent configurations!
- Entanglement entropy of boundary region A is given by the area of bulk spacetime extremal surface E

- Such extremal surface can traverse across time;
  e.g. for a rotating black hole in AdS:

extremal surface E



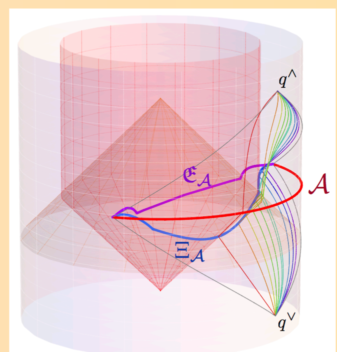
Implications

- Hence even though entanglement entropy is a rather complex and mysterious object, in the holographic dual language it is mapped to a very simple geometrical construct: extremal surface.
- This hints at a profound connection between spacetime in the bulk and entanglement structure on the boundary.
- Through holography, we learn that classical geometry secretly knows about quantum mechanics!

Subsequent lessons

- Satisfies non-trivial causality constraints.
- Encodes bulk spacetime geometry.
- Provides fine-grained information about the state (e.g. if it is pure or mixed).
- Intriguing connections to 'causal holographic information'

e.g. collapsing black hole:
extremal surface
causal information surface
provide different measures of quantum information



Practical applications

- This holographic prescription allows us to study entanglement in regimes where all currently known field theory techniques fail.
- It also enables us to prove important properties of entanglement, and uncover previously unknown ones.

Uses of holography

- Unites many subfields of physics
- Holographic description provides the only known tool for calculating Entanglement Entropy in general settings, as well as many other important features of strongly interacting systems
- Geometrizes fundamental quantities => new perspective

Uses of entanglement

- Provides new puzzles & new directions of research
- Elucidates the AdS/CFT map, & especially the emergence of spacetime
- Hints at the lesson [Cf. Van Raamsdonk, Susskind, Maldacena,...] that entanglement builds bridges
  (ER = EPR: Einstein-Rosen bridge via Einstein-Podolsky-Rosen entanglement)