"dlog" Forms & Differential Equations

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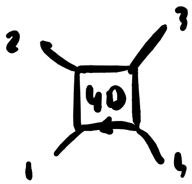
Motivation

- Integrands in planar & non-planar $\mathcal{N}=4$ SYM
  $\longrightarrow$ forms with logarithmic singularities
- Interesting geometry behind (Amplituhedron...)
- Big Q: What does this imply for
  integrated amplitudes?
- Modest Q: What use can we make of
  nice "dlog" forms?

Two types of "dlogs"

- "dlog" integrands



$$= \int d\log \frac{\ell^2}{(\ell-\ell_*)^2} \wedge d\log \frac{(\ell-k_1)^2}{(\ell-\ell_*)^2} \wedge d\log \frac{(\ell-k_{12})^2}{(\ell-\ell_*)^2} \wedge d\log \frac{(\ell+k_4)^2}{(\ell-\ell_*)^2}$$

ℓ_* cut solution : $\ell_*^2 = (\ell_* - k_1)^2 = (\ell_* - k_{12})^2 = (\ell_* - k_4)^2 = 0$

- "dlog" differential equations

$$dI_i = \epsilon \, d\log \alpha_i \, A_{ij} \, I_j$$

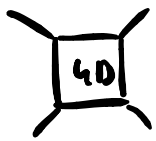
master integrals

Require solving IPB → large linear systems !

$$\text{dlogs} \xrightarrow{?} \text{dlogs}$$

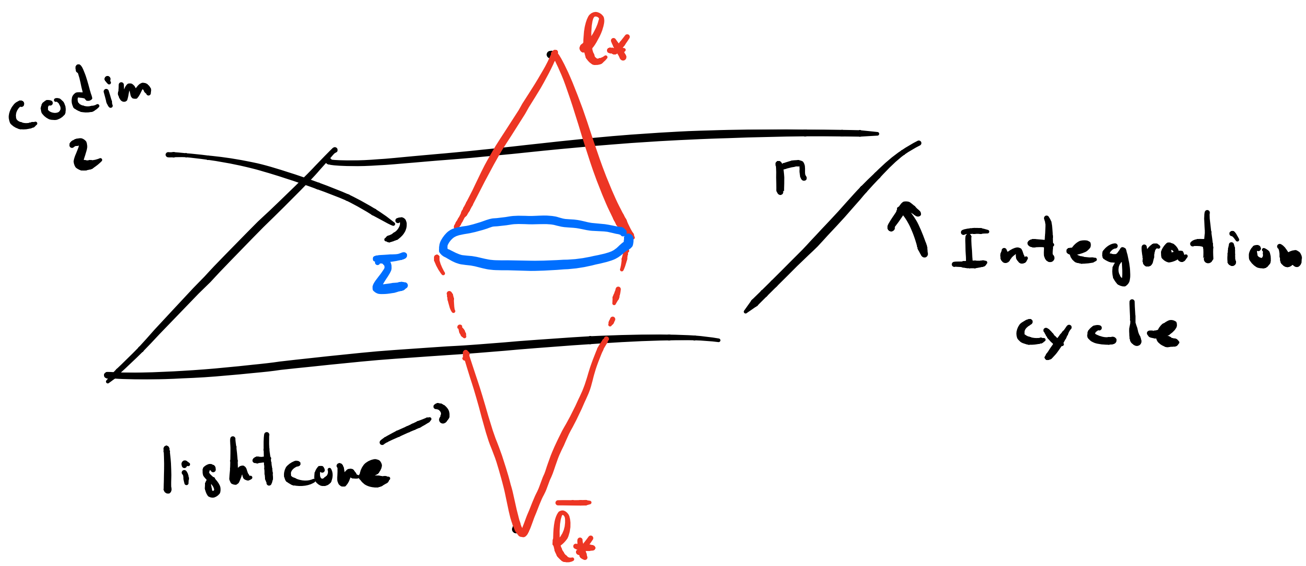
"dlog" Localization

Most "dlog" forms are not exact

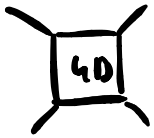


$$\neq \int_{\Gamma} d(\quad) \longleftrightarrow d^2 \log(\ell-\ell_*)^2 = \delta\left((\ell-\ell_*)^2\right)$$

contact terms localize integral

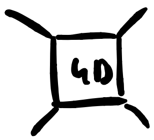


- Non-rational localized integrand



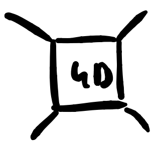
$$= \int_{\Sigma} \log(\cdot\cdot)\, d\log(\cdots) \wedge d\log(\cdot\cdot)$$

- Take derivative to make rational

d 

$$= \int_{\Sigma} d\log(\cdot\cdot)\, d\log(\cdots) \wedge d\log(\cdot\cdot) + \ldots$$

- Partial fraction (a.k.a unitarity)

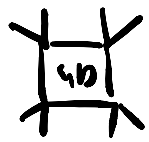
d 

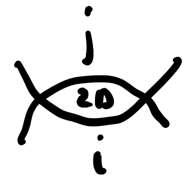
$$= d\log \alpha_{12} \int_{\Sigma} d\log(\cdots) \wedge \underbrace{d\log(\cdot\cdot)}_{D=2 \text{ bubble}} + \ldots$$

- Works at one loop [Caron-Huot]

$$d\begin{pmatrix} n\text{-gon} \\ n\text{-dim} \end{pmatrix} = \sum d\log \alpha_i \begin{pmatrix} n\text{-2 gon} \\ n\text{-2 dim} \end{pmatrix}_i$$

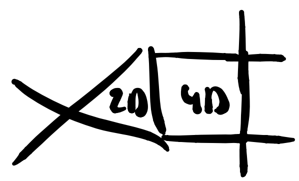
eg

d 

$= \sum_{i<j} d\log \alpha_{ij}$ 

- And also at higher loops [Herrmann, JPM]

d 

$= d\log \alpha_1$ 

$+ \cdots$

- Residue theorems $\to$ minimal alphabet
- No large linear systems!!

Lots to do!

- Extension to non-finite integrals
- Extension to "dElliptic"?
- Turn finding "dlog" forms from an art into a science
- Application to interesting unknown integrals (e.g. fishnet)
- Application to full amplitudes??

THANK YOU!!!