# The planar double box integral for top pair production with a closed top loop to all orders in the dimensional regularisation parameter 

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Based on arXiv: 1804.11144 by Luise Adams, Ekta Chaubey and Stefan Weinzierl.

## Introduction



- The planar double box enters the next-to-next-to-leading order (NNLO) contribution for process $p p \rightarrow t \bar{t}$.
- Depends on 2 scales and contains the sunrise graph as a subtopology.
- Naturally a case of elliptic generalisations of multiple polylogarithms.
- Presence of not one but three elliptic curves.
- System of differential equations brought to a form linear in $\epsilon$, where
$\epsilon^{0}$ part is strictly lower triangular.
- Transformation of the basis of master integrals rational in $x, y$, the periods of the three elliptic curves and their $y$-derivatives.


## Differential Equations (D.E.) system linear in $\epsilon$

- Changing the basis, $\vec{J}=U \vec{I}$ D.E. transforms into

$$
d \vec{J}=A^{\prime} \vec{J}, \quad A^{\prime}=U A U^{-1}+U d U^{-1} .
$$

- There exists a transformation U, s.t. $A^{0}$ is strictly lower triangular

$$
d \vec{J}=\left(A^{(0)}(x, y)+\epsilon A^{(1)}(x, y)\right) \vec{J} .
$$

- Transformation matrix rational in

$$
\epsilon, x, y, \psi_{1}^{(a)}, \psi_{1}^{(b)}, \psi_{1}^{(c)}, \partial_{y} \psi_{1}^{(a)}, \partial_{y} \psi_{1}^{(b)}, \partial_{y} \psi_{1}^{(c)} .
$$

## An example

- The 3 master integrals can be taken as

$$
\begin{aligned}
& J_{24}=\epsilon^{3} \frac{\left(1+x^{2}\right)^{2}}{x\left(1-x^{2}\right)} \frac{\pi}{\psi_{1}^{(b)}} I_{1112001} \\
& J_{25}=\epsilon^{3}(1-2 \epsilon) \frac{\left(1+x^{2}\right)^{2}}{x\left(1-x^{2}\right)} I_{1111001}+R_{25,24} \frac{\psi_{1}^{(b)}}{\pi} J_{24}, \\
& J_{26}=\frac{6}{\epsilon} \frac{\left(\psi_{1}^{(b)}\right)^{2}}{2 \pi i W_{y}^{(b)}} \frac{d}{d y} J_{24}+R_{26,24}\left(\frac{\psi_{1}^{(b)}}{\pi}\right)^{2} J_{24}-\frac{\epsilon^{2}}{24}\left(y^{2}-30 y-27\right) \frac{\psi_{1}^{(b)}}{\pi} D^{-} I_{1001001}
\end{aligned}
$$



- This pattern applies to all the elliptic sectors.
- $A^{0}$ vanishes for $\mathrm{x}=0$ or $\mathrm{y}=1$.
- For $\mathrm{y}=1$, entries of $A^{1}$ reduce to differential one-forms, for $\mathrm{x}=0$ they reduce to modular forms.


## Thank you!

