Scattering amplitudes

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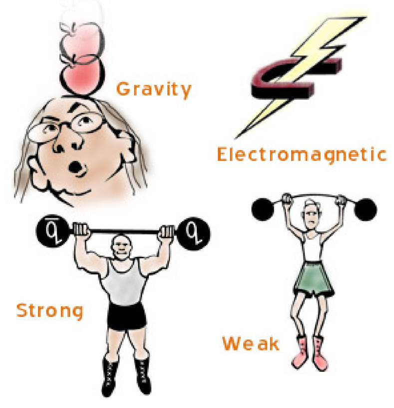
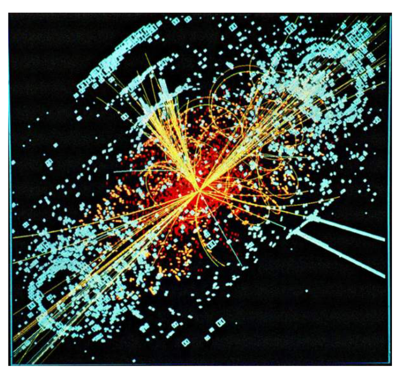
I. Particle Interactions

Our world is made of elementary particles.



- Constituents of matter:
  Leptons (electrons), quarks.
- Mediate fundamental forces:
  Photons, gluons, W and Z bosons, gravitons.



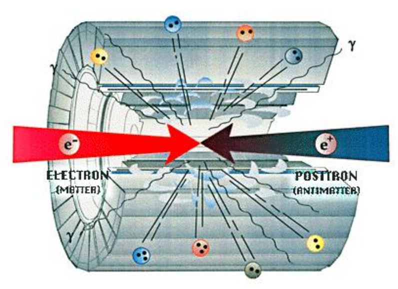
Particle colliders

⋆ Particles accelerated and smashed together.
⋆ Result: a spray of new particles.
⋆ Analysis: discovery of particles and their properties.

In 2013 LHC discovered Higgs boson, last piece of the Standard Model.

Scattering process of elementary particles:

- Interaction of two or more particles.
- Fixed initial states, final states can vary.
- Fundamental laws reveal in these processes.



Mathematical description: **scattering amplitude**.

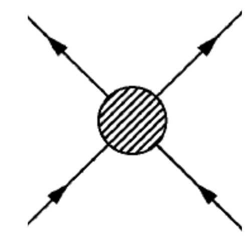
⋆ Describes a probability that a given scattering process happens.
⋆ It is a function of momenta and spins, $\mathcal{M}(p,s)$.

II. Quantum Field Theory

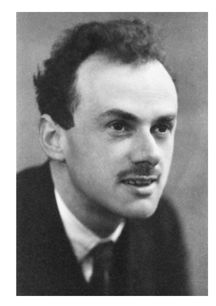
Theoretical framework for predicting scattering amplitudes.

Consistent with principle of special relativity and quantum mechanics.

- **Locality**: All interactions are pointlike.
- **Unitarity**: For all possible outcomes of a scattering process the probabilities must sum to one, $\sum_j p_j = 1$.



Quantum field theory (QFT) is specified by a set of properties



Paul Dirac, first pioneer of QFT

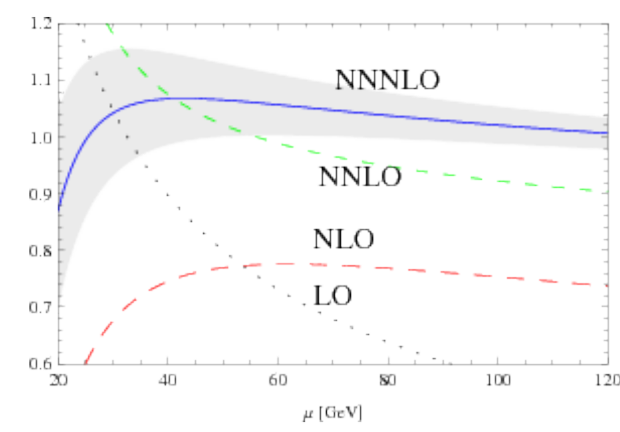
⋆ Particle content: each described by a field ϕ, ψ, A_μ.
⋆ Symmetries of the theory.
⋆ Interactions between fields given by Lagrangian $\mathcal{L}$.
⋆ The strength of the interaction: coupling constants g.

Perturbative expansion of scattering amplitudes

- **Weak coupling**: expansion around $g = 0$,

$$\mathcal{M} = g\mathcal{M}_1 + g^2\mathcal{M}_2 + g^3\mathcal{M}_3 + \dots$$

- Each contribution $\mathcal{M}_j$ can be calculated from the Lagrangian in perturbation theory.
- Graphical picture: **Feynman diagrams**.

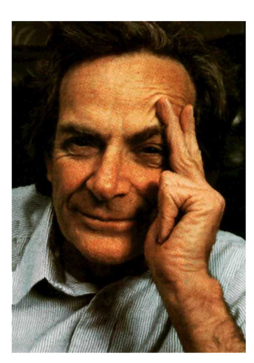


Different orders in perturbation theory
LO = leading order, 'N' stands for 'next'

III. Feynman Diagrams

Universal diagrammatic approach

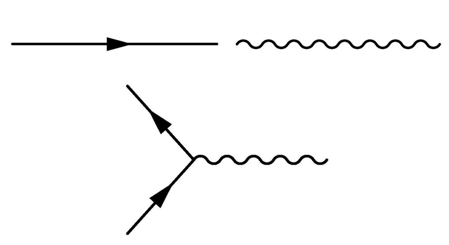
⋆ Simple organization of the perturbation expansion.
⋆ Calculate scattering amplitude for a given process
  = Rewrite it as a sum of building blocks

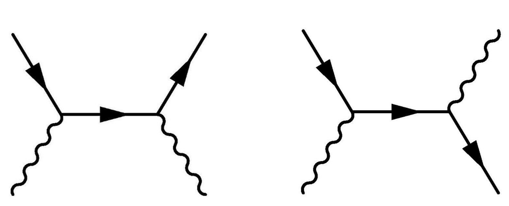


Richard Feynman

Calculation using Feynman diagrams

- Each term in the Lagrangian can be represented as a line or a vertex.
- Draw all possible diagrams from them.

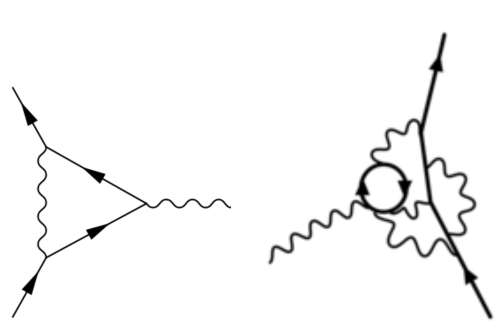




∗ Incoming and outgoing particles fixed.
∗ Rules for writing a formula for diagram.

Perturbative expansion = loop expansion of Feynman diagrams.

- Diagrams with internal loops: higher powers of g.
- They are higher order terms in perturbation theory.
- They should be suppressed in 'good' theories.



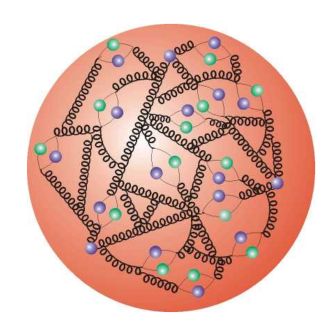
Great universal approach to quantum field theory!

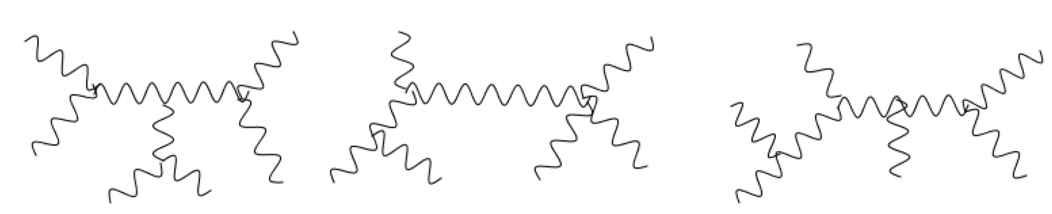
Problem: Huge cancelations between diagrams, some properties **invisible**.

IV. Hidden Structures

First evidence for hidden structures: six gluon scattering.

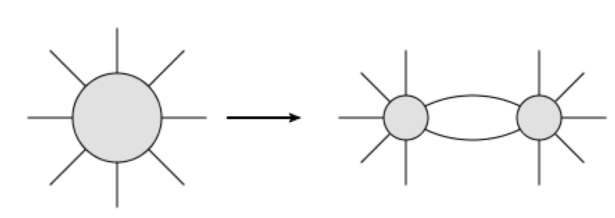
- Gluons confined inside the proton.
- At high energies the gluon scattering dominates.
- Calculated in 1985 for the new planned collider SSC.
- 220 Feynman diagrams, 100 pages of result.





The final result shrinks to $\mathcal{M} = \dfrac{\langle 12\rangle^3}{\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 61\rangle}$ where $\langle ab\rangle$ are related to momenta.

Methods to calculate scattering amplitudes without Feynman diagrams.



Unitarity cut: amplitude factorizes to two pieces for special kinematics

⋆ Use consistency conditions to fix amplitudes.
⋆ Amplitude = It is an unique function consistent with principles locality and unitarity.
⋆ Powerful tools: unitarity cuts, recursion relations...

Revolution in last 10 years: hidden mathematical structures.

- Huge advance in calculations using computers.
- New techniques: integrability, twistor strings, twistors.
- Geometric definition: amplitudes are **volumes**!