Dual Conformal Structure Beyond The Planar Limit

QMAP
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Motivation

Planar N=4 SYM is important

- Dual Conformal Symmetry (hidden)
- Integrability
- Yangian Symmetry
- Wilson Loop Duality
- Uniform Transcendentality
- Amplituhedron

Could these properties hold in the non-planar sector?

$$\mathcal{M}_{12\ldots n} = \mathrm{PT}(123\ldots n) \int \mathcal{I}$$

- Logarithmic Singularities
- Unit Leading Singularities
- No Poles At Infinity

2-loop 4- and 5-points
3-loop 4-points

YES

Bern, Herrmann, Litsey, Stankowicz, Trnka
1412.8584
1512.08591

$$\mathcal{M} = \sum_{k,\sigma,j} a_{\sigma,k,j} c_k \mathrm{PT}_\sigma \int \mathcal{I}^j$$

- Logarithmic Singularities
- Unit Leading Singularities
- No Poles At Infinity

Planar

Hidden Symmetry

Analytic Properties

?

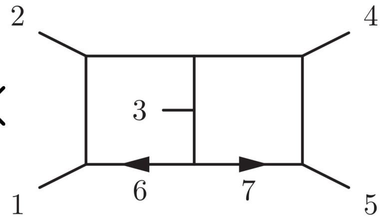
Non-Planar ✓

Hidden Symmetry?

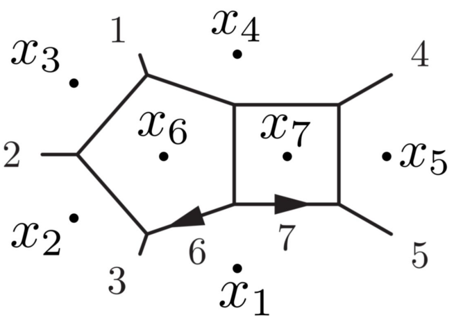
Nonplanar hidden symmetries

N

$$\langle 13\rangle\langle 24\rangle\Big[[24][13]\left(\ell_7+\tfrac{[45]}{[24]}\lambda_5\widetilde{\lambda}_2\right)^2\left(\ell_6-\tfrac{Q_{12}\cdot\widetilde{\lambda}_3\ \widetilde{\lambda}_1}{[13]}\right)^2 - [14][23]\left(\ell_7+\tfrac{[45]}{[14]}\lambda_5\widetilde{\lambda}_1\right)^2\left(\ell_6-\tfrac{Q_{12}\cdot\widetilde{\lambda}_3\ \widetilde{\lambda}_2}{[23]}\right)^2\Big] \times$$



$$= \int d^D l_6 d^D l_7 \frac{N}{\prod_k \rho_k} = \int \mathcal{I}$$



$$\rho_1 = (x_6 - x_1)^2 \qquad \rho_2 = (x_6 - x_3 + p_3)^2$$
$$\rho_3 = (x_6 - x_4 + p_3)^2 \qquad \rho_4 = (x_7 - x_4)^2$$
$$\rho_5 = (x_7 - x_5)^2 \qquad \rho_6 = (x_7 - x_1)^2$$
$$\rho_7 = (x_6 - x_7)^2 \qquad \rho_8 = (x_6 - x_7 + p_3)^2$$

$$\delta x_i = \frac{1}{2}x_i^2 b^\mu - (b\cdot x_i)x_i^\mu \Rightarrow \frac{\delta(x_i - x_j)^2}{(x_i - x_j)^2} = -b\cdot(x_i + x_j) = \frac{\delta(x_i - x_j \pm p_k)^2}{(x_i - x_j \pm p_k)^2}$$ if $b^\mu \propto p_k^\mu$

So all propagators transform as $\delta\rho_k \propto \rho_k$

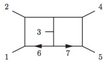
Quite nontrivially, we also have $\delta N = \delta\Big(\langle 13\rangle\langle 24\rangle\Big[[24][13]\Big(\ell_7 + \frac{[45]}{[24]}\lambda_5\tilde\lambda_2\Big)^2\Big(\ell_6 - \frac{Q_{12}\cdot\tilde\lambda_3\ \tilde\lambda_1}{[13]}\Big)^2 - [14][23]\Big(\ell_7 + \frac{[45]}{[14]}\lambda_5\tilde\lambda_1\Big)^2\Big(\ell_6 - \frac{Q_{12}\cdot\tilde\lambda_3\ \tilde\lambda_2}{[23]}\Big)^2\Big]\Big) \propto N$

with the proportionality factor being just that to make

$$\delta\mathcal{I} = -(D-4)[b\cdot(l_5+l_6)]\mathcal{I}$$

Thus, this is a hidden symmetry when $D=4$.

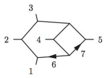
In fact, all nonplanar 2-loop 5-pt integrands

(a) 

$$N_1^{(a)} = \langle 13\rangle\langle 24\rangle\Big[[24][13]\Big(\ell_7 + \frac{[45]}{[24]}\lambda_5\tilde\lambda_2\Big)^2\Big(\ell_6 - \frac{Q_{12}\cdot\tilde\lambda_3\ \tilde\lambda_1}{[13]}\Big)^2 - [14][23]\Big(\ell_7 + \frac{[45]}{[14]}\lambda_5\tilde\lambda_1\Big)^2\Big(\ell_6 - \frac{Q_{12}\cdot\tilde\lambda_3\ \tilde\lambda_2}{[23]}\Big)^2\Big],$$

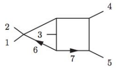
$$N_2^{(a)} = N_1^{(a)}\big|_{\substack{1\leftrightarrow 2\\ 4\leftrightarrow 5}},\quad N_3^{(a)} = N_1^{(a)}\big|_{\substack{2\leftrightarrow 4\\ 1\leftrightarrow 5}},\quad N_4^{(a)} = N_1^{(a)}\big|_{\substack{1\leftrightarrow 4\\ 2\leftrightarrow 5}},$$

$$N_5^{(a)} = \overline{N}_1^{(a)},\quad N_6^{(a)} = \overline{N}_2^{(a)},\quad N_7^{(a)} = \overline{N}_3^{(a)},\quad N_8^{(a)} = \overline{N}_4^{(a)},$$

c) 

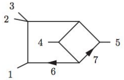
$$N_1^{(c)} = [13]\Big(\ell_6 + \frac{Q_{45}\cdot\tilde\lambda_3\ \tilde\lambda_1}{[13]}\Big)^2\langle 15\rangle[54]\langle 43\rangle(\ell_6+k_4)^2,$$

$$N_2^{(c)} = N_1^{(c)}\big|_{4\leftrightarrow 5},\quad N_3^{(c)} = \overline{N}_1^{(c)},\quad N_4^{(c)} = \overline{N}_2^{(c)},$$

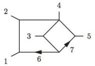
(d) 

$$N_1^{(d)} = s_{34}(s_{34}+s_{35})\Big(\ell_7 - k_5 + \frac{\langle 35\rangle}{\langle 34\rangle}\lambda_4\tilde\lambda_5\Big)^2,$$

$$N_2^{(d)} = N_1^{(d)}\big|_{4\leftrightarrow 5},\quad N_3^{(d)} = \overline{N}_1^{(d)},\quad N_4^{(d)} = \overline{N}_2^{(d)},$$

(f) 

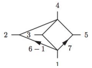
$$N_1^{(f)} = s_{14}s_{45}(\ell_6+k_5)^2,\qquad N_2^{(f)} = N_1^{(f)}\big|_{4\leftrightarrow 5},$$

(h) 

$$N_1^{(h)} = \langle 15\rangle[35]\langle 23\rangle[12]\Big(\ell_6 - \frac{\langle 12\rangle}{\langle 32\rangle}\lambda_3\tilde\lambda_1\Big)^2,\quad N_2^{(h)} = N_1^{(h)}\big|_{3\leftrightarrow 5},$$

$$N_3^{(h)} = s_{12}\langle 13\rangle[15]\langle 5|\ell_6|3],\quad N_4^{(h)} = s_{12}[13]\langle 15\rangle\langle 3|\ell_6|5],$$

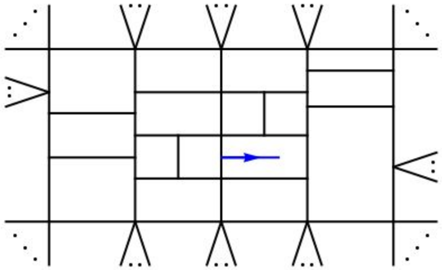
$$N_5^{(h)} = \overline{N}_1^{(h)},\quad N_6^{(h)} = \overline{N}_2^{(h)},$$

(i) 

$$N_1^{(i)} = \langle 2|4|3]\langle 3|5|2] - \langle 3|4|2]\langle 2|5|3].$$

have an analogous hidden symmetry when $D=4$.

Identity on previous slide allows for infinite class of nonplanar integrals:



Future

- Other topologies?
- More symmetries?
- Predict (parts of) amplitudes?
- Differential equations?

Thank you!