Spinor-helicity and exact solutions of the Einstein equations

Izzy Nicholson
University of Edinburgh
i.nicholson@sms.ed.ac.uk

Higher dimensional black hole classification from spinor helicity

Based on work done with Donal O'Connell and Ricardo Monteiro

$$SO(3,1) \cong SL(2,\mathbb{C})/\mathbb{Z}_2$$

$$\downarrow$$

$$p \cdot \sigma_{a\dot{a}} = \lambda_a \, \tilde{\lambda}_{\dot{a}}$$

$$q \cdot \sigma_{a\dot{a}} = \mu_a \, \tilde{\mu}_{\dot{a}}$$

Normal spinor helicity…

See eg Elvang & Huang (2013)

…can be applied to GR

$$\varepsilon_+^\mu = \sigma^\mu{}_{a\dot{a}}\, \lambda^a\, \tilde{\mu}^{\dot{a}}, \qquad \varepsilon_-^\mu = \left(\varepsilon_+^\mu\right)^*$$

This is the Newman-Penrose formalism - Penrose (1960), Newman & Penrose (1962)

…can be applied to GR

$$\varepsilon_+^\mu = \sigma^\mu{}_{a\dot{a}}\,\lambda^a\,\tilde{\mu}^{\dot{a}}, \qquad \varepsilon_-^\mu = \left(\varepsilon_+^\mu\right)^*$$

$$g^{\mu\nu} = p^\mu q^\nu + p^\nu q^\mu - \varepsilon_+^\mu\,\varepsilon_-^\nu - \varepsilon_+^\nu\,\varepsilon_-^\mu$$

This is the Newman-Penrose formalism - Penrose (1960), Newman & Penrose (1962)

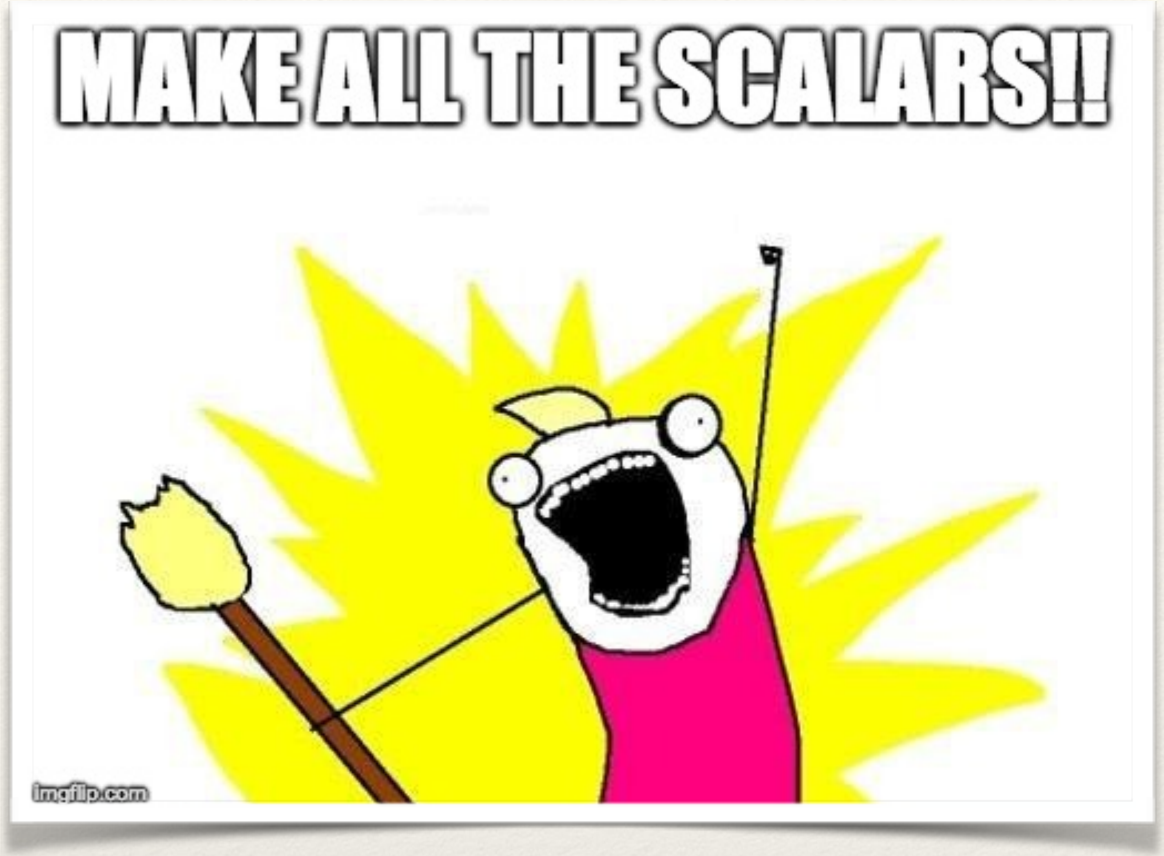
...can be applied to GR

$$\varepsilon_+^\mu = \sigma^\mu{}_{a\dot{a}}\,\lambda^a\,\tilde{\mu}^{\dot{a}}, \qquad \varepsilon_-^\mu = \left(\varepsilon_+^\mu\right)^*$$

$$g^{\mu\nu} = p^\mu q^\nu + p^\nu q^\mu - \varepsilon_+^\mu\,\varepsilon_-^\nu - \varepsilon_+^\nu\,\varepsilon_-^\mu$$

$$\boxed{\Psi_{abcd} = C_{\mu\nu\rho\sigma}\,\sigma^{\mu\nu}{}_{ab}\,\sigma^{\rho\sigma}{}_{cd}}$$

This is the Newman-Penrose formalism - Penrose (1960), Newman & Penrose (1962)



$$\psi_0 = \Psi_{abcd}\,\lambda^a\lambda^b\lambda^c\lambda^d$$

$$\psi_1 = \Psi_{abcd}\,\lambda^a\lambda^b\lambda^c\mu^d$$

$$\vdots \qquad \vdots$$

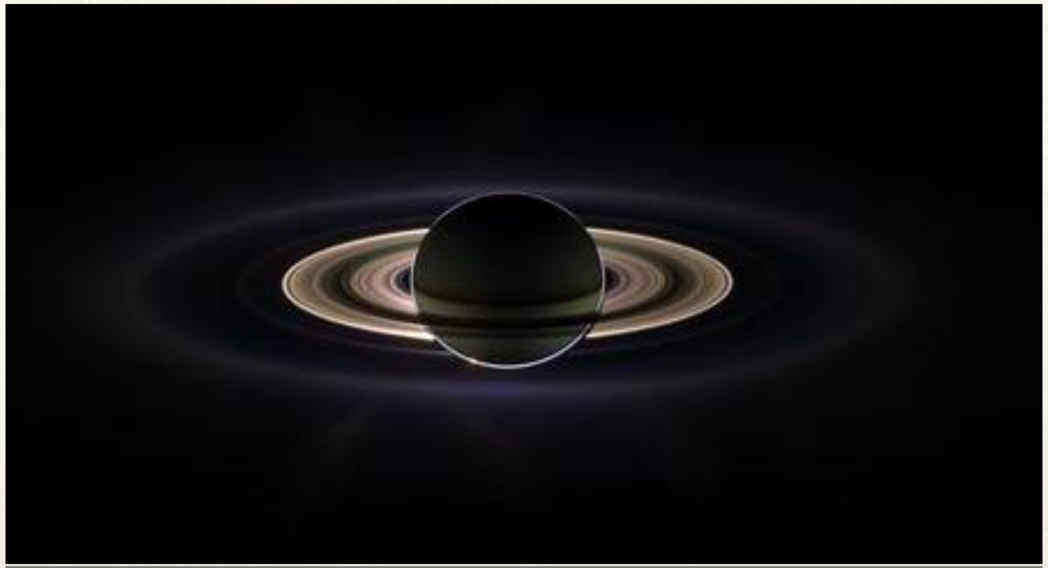
$$D\sigma - \delta\kappa = (\rho + \bar{\rho} + 3\epsilon - \bar{\epsilon})\sigma - (\tau - \bar{\pi} + \bar{\alpha} + 3\beta)\kappa + \psi_0$$

$$\vdots \qquad \vdots \qquad \vdots$$

It's very useful!

Petrov (1954)
Kinnersley (1969)

Why higher D?



See eg Emparan & Reall (2008)
Elvang & Figueras (2007)

~~Uniqueness~~

de Smet (2002)
Coley, Milson et al (2004, 5)
Gobez-Lobo & Martin-Garcia (2009)

$$SO(4,1) \cong Sp^*(1,1)/\mathbb{Z}_2$$

But 5D spinor-helicity is easy…

Cheung & O'Connell (2009)

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$$\gamma^\mu{}_{AB}$$

↑

$$A, B = 1, \ldots, 4$$

But 5D spinor-helicity is easy…

Cheung & O'Connell (2009)

$$SO(4,1) \cong Sp^*(1,1)/\mathbb{Z}_2$$

$$\gamma^{\mu}{}_{AB}$$

$$A, B = 1, \ldots, 4$$

$$\mathfrak{so}(3) \cong \mathfrak{su}(2)$$

$$p \cdot \gamma_{AB} = \lambda_A{}^a \lambda_B{}^b \epsilon_{ab}$$

But 5D spinor-helicity is easy…

Cheung & O'Connell (2009)

So we apply it to GR again:

$$p \cdot \gamma_{AB} = {\lambda_A}^a {\lambda_B}^b \epsilon_{ab}$$

$$q \cdot \gamma_{AB} = {\mu_A}^a {\mu_B}^b \epsilon_{ab}$$

So we apply it to GR again:

$$p \cdot \gamma_{AB} = \lambda_A{}^a \lambda_B{}^b \, \epsilon_{ab}$$

$$q \cdot \gamma_{AB} = \mu_A{}^a \mu_B{}^b \, \epsilon_{ab}$$

$$\varepsilon^\mu{}_{ab} = \gamma^\mu{}_{AB} \, \lambda^A{}_a \, \mu^B{}_b$$

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$$g^{\mu\nu} = p^\mu q^\nu + p^\nu q^\mu - \epsilon^{ac} \epsilon^{bd} \, \varepsilon^\mu{}_{ab} \, \varepsilon^\nu{}_{cd}$$

So we apply it to GR again:

$$p \cdot \gamma_{AB} = \lambda_A{}^a \lambda_B{}^b \, \epsilon_{ab}$$

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$$\varepsilon^\mu{}_{ab} = \gamma^\mu{}_{AB} \, \lambda^A{}_a \, \mu^B{}_b$$

$$g^{\mu\nu} = p^\mu q^\nu + p^\nu q^\mu - \epsilon^{ac} \epsilon^{bd} \, \varepsilon^\mu{}_{ab} \, \varepsilon^\nu{}_{cd}$$

$$\Psi_{ABCD} = C_{\mu\nu\rho\sigma} \, \gamma^{\mu\nu}{}_{AB} \, \gamma^{\rho\sigma}{}_{CD}$$

$$\psi^{(0)}_{abcd} = \Psi_{ABCD}\, \lambda^{A}{}_{a}\, \lambda^{B}{}_{b}\, \lambda^{C}{}_{c}\, \lambda^{D}{}_{d}$$

$$\psi^{(1)}_{abcd} = \Psi_{ABCD}\, \lambda^{A}{}_{a}\, \lambda^{B}{}_{b}\, \lambda^{C}{}_{c}\, \mu^{D}{}_{d}$$

$$\vdots \qquad\qquad \vdots$$

$$\psi^{(1)}_{abcd} = \psi^{(1)}_{a(bcd)} \rightarrow \psi^{(1)} = \psi^{(1)}_{\mathrm{SYM}} + \epsilon \otimes \chi^{(1)}$$

And obtain little group spinors

4D		5D		
ψ_0		$\psi^{(0)}$		
ψ_1		$\psi^{(1)}$	$\chi^{(1)}$	
ψ_2	**vs**	$\psi^{(2)}$	$\chi^{(2)}$	$\psi^{(2)}_{\mathrm{tr}}$
ψ_3		$\psi^{(3)}$	$\chi^{(3)}$	
ψ_4		$\psi^{(4)}$		

Thank you!