# Geometry and physics in Theory $\mathcal{X}$

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*the 6d $\mathcal{N}=(2,0)$ theory*

## The 6d (2,0) theory ($\mathcal{X}$)

is really a family of QFT's, labelled by a symmetry algebra g of type ADE: su(N), so(2N), $e_6$, $e_7$, $e_8$.

These quantum field theories are:

- conformal (scale-invariant & more)
- strongly coupled (w/ no tunable parameters)
- maximally supersymmetric
- non-Lagrangian! (no fields, action, path integral...)

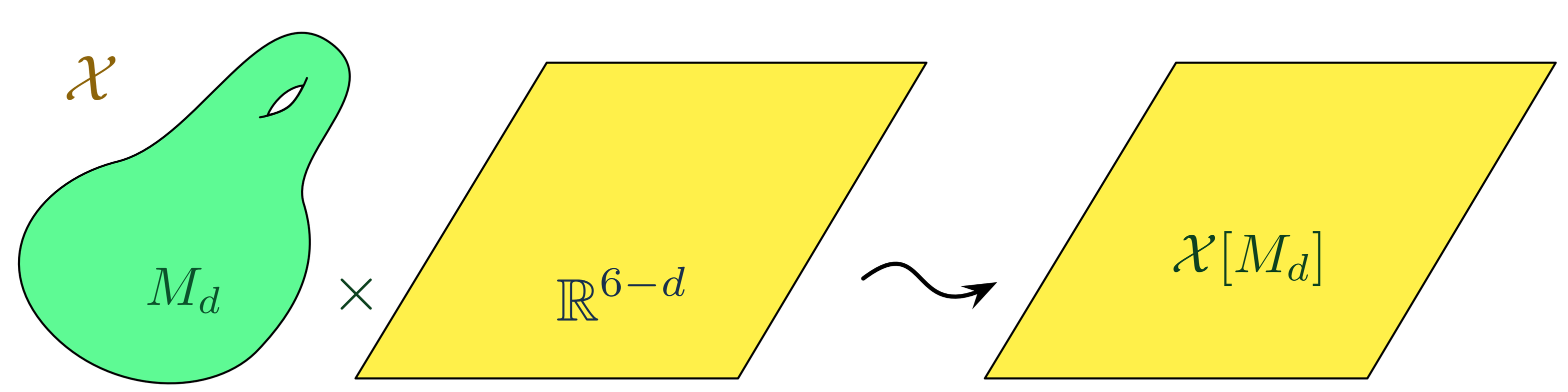
*i.e. interesting and difficult*

Nevertheless, there is much evidence for their existence, from string/M-theory, from consitency of compactifications, and from direct calculations (conformal bootstrap).
[Strominger, Witten ~'95]

Huge open problem:
Find all the operators (local & nonlocal) in $\mathcal{X}$ and their algebra/correlation functions!

## Compactification

Take 6d spacetime of the form $M_d \times \mathbb{R}^{6-d}$
(add background fields on $M_d$ to preserves supersymmetry - there are choices)




effective QFT in (6-d) dimensions

The effective theory depends on much less than the full metric of $M_d$, *e.g.* only its conformal structure or even just its topology!

So, we get field theories $\mathcal{X}[M_d]$ whose physics is controlled by the topology/geometry of a d-manifold $M_d$.

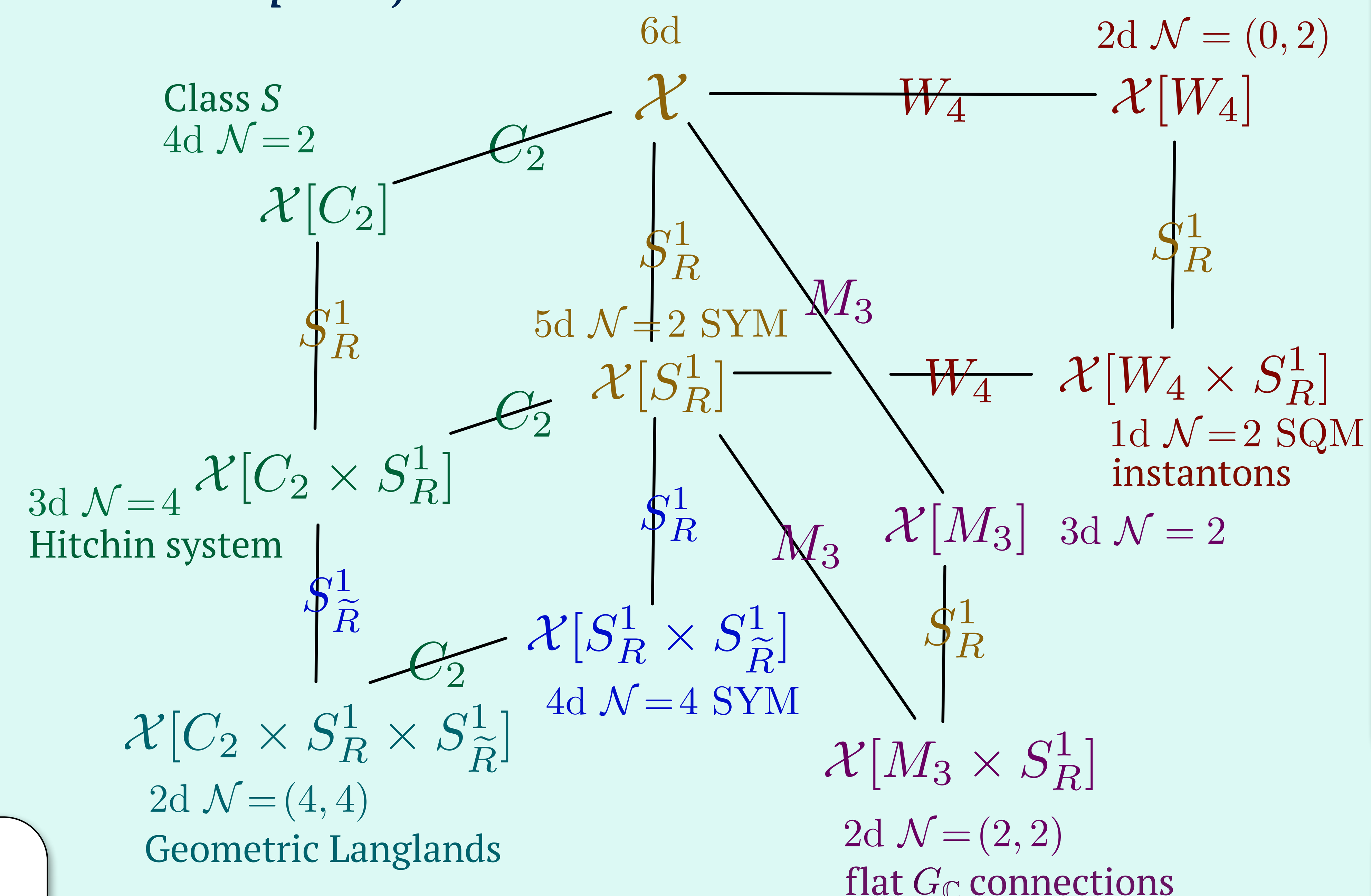
→ find new, powerful invariants of d-manifolds, that quantize and categorify classical invariants
*e.g. hyperbolic volumes, moduli spaces of flat connections, ...*

→ can apply geometric intuition to (6-d)-dimensional physics (putting string/M-theory on Calabi-Yau has the same flavor)

→ gluing manifolds from building blocks (handles, simplices) translates to "gluing" the content of QFT's
glue ↔ gauge global symmetries, add interactions...

different ways to glue the same $M_d$ ↔ dual descriptions of $\mathcal{X}[M_d]$

## *some compactifications...*




## 5d super-Yang-Mills (SYM)

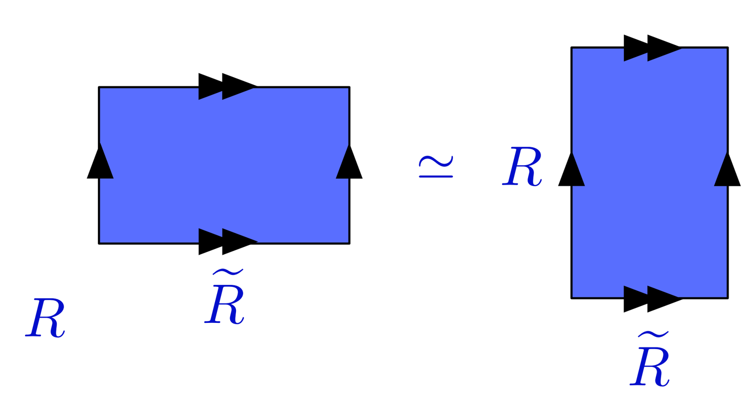
$\mathcal{X}[S^1_R]$ is 5d maximally supersymmetric Yang-Mills theory, with gauge group G = SU(N), SO(2N), $E_6$, $E_7$, or $E_8$

- ordinary, Lagrangian theory
- gauge coupling strength $g^2_{5d} = R$ (radius)

## 4d super-Yang-Mills (SYM) $\mathcal{X}[S^1_R \times S^1_{\widetilde{R}}]$

Adding a second circle leads to 4d maximally SUSY'c Yang-Mills, with gauge coupling $g^2_{4d} = \frac{g^2_{5d}}{\widetilde{R}} = \frac{R}{\widetilde{R}}$

Two ways to view the same torus predicts a highly nontrivial symmetry




$\mathcal{X}[S^1_R \times S^1_{\widetilde{R}}] \leftrightarrow \mathcal{X}[S^1_{\widetilde{R}} \times S^1_R]$

$g^2_{4d} \leftrightarrow \frac{1}{g^2_{4d}}$

4d electric-magnetic duality!
[Montonen-Olive '77]

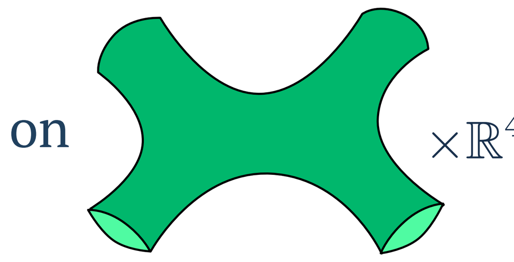
## What is $\mathcal{N}$ ?

A measure of how much supersymmetry (SUSY) there is.
SUSY is a global symmetry that relates bosons and fermions.
Roughly, $\mathcal{N}$ = # of spinors of $SO(d-1,d)_{\text{Lorentz}}$ that generate SUSY.

## 4d $\mathcal{N}=2$ theories $\mathcal{X}[C_2]$ (Seiberg-Witten type)

from a general 2d surface $C_2$

*E.g.*
6d su(2) theory on

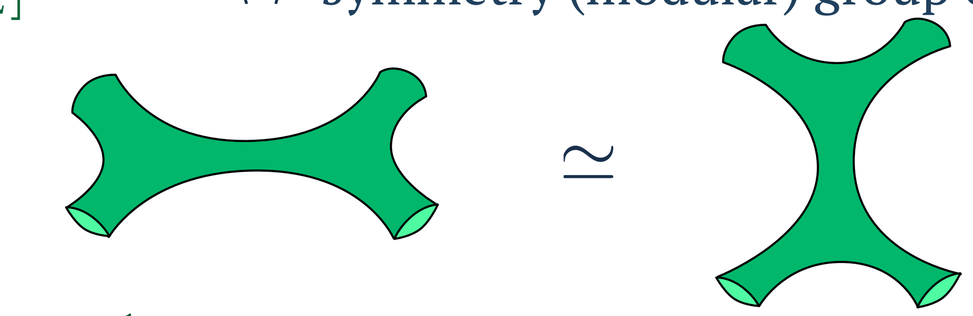


$\times \mathbb{R}^4$ ⇝ 4d SU(2) (SUSY-)QCD w/ 4 flavors of matter

[Gaiotto '09]

Gauge couplings of $\mathcal{X}[C_2]$ (and other marginal parameters) ↔ shape (conformal structure) of $C_2$

Duality group of $\mathcal{X}[C_2]$ ↔ symmetry (modular) group of $C_2$

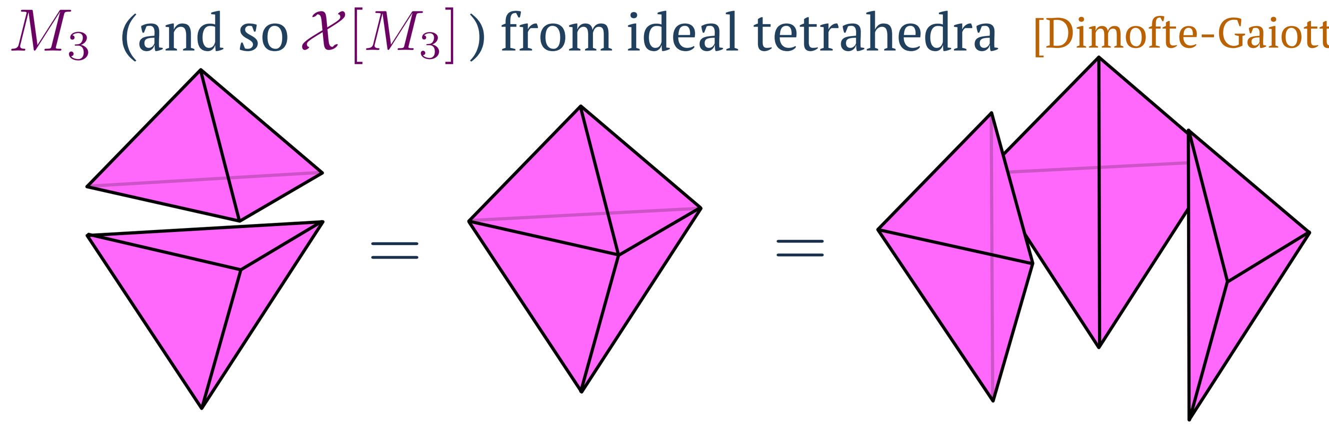



Add a circle:
vacua of $\mathcal{X}[C_2 \times S^1_R]$ ↔ sol's of Hitchin's equations on $C_2$
(use physics to construct hyperkahler metric on this space!)
[Gaiotto-Moore-Neitzke '08]

## 3d $\mathcal{N}=2$ theories from 3-manifolds $\mathcal{X}[M_3]$

Depend only on hyperbolic (or other uniformized) structure on $M_3$
often unique ⇒ $\mathcal{X}[M_3]$ is a topological invariant!
[Mostow '73, Thurston 80's,...]

Glue $M_3$ (and so $\mathcal{X}[M_3]$) from ideal tetrahedra [Dimofte-Gaiotto-Gukov '11]




3d QED w/ 2 flavors ≃ 3 bosons+fermions with cubic coupling

Vacua of $\mathcal{X}[M_3 \times S^1_R]$ ↔ flat $SL(N,\mathbb{C})$ (etc) connections on $M_3$

Partition function of $\mathcal{X}[M_3]$ on Lens space $S^3/\mathbb{Z}_k$ ↔ invariant of $M_3$

Hilbert space of $\mathcal{X}[M_3]$ on $S^2 \times \mathbb{R}_{\text{time}}$ ↔ categorified hyperbolic geometry?

Lots to understand!

## 4d $\mathcal{N}=(0,2)$ theories $\mathcal{X}[W_4]$

are invariants of 4-manifolds [Gadde-Gukov-Putrov '14]

Vacua of $\mathcal{X}[W_4 \times S^1_R]$ ↔ instantons $(F = *F)$ on $W_4$

Partition function of $\mathcal{X}[W_4]$ ↔ Vafa-Witten invariants of $W_4$ (Euler characters of $\mathcal{M}_{\text{instanton}}$)