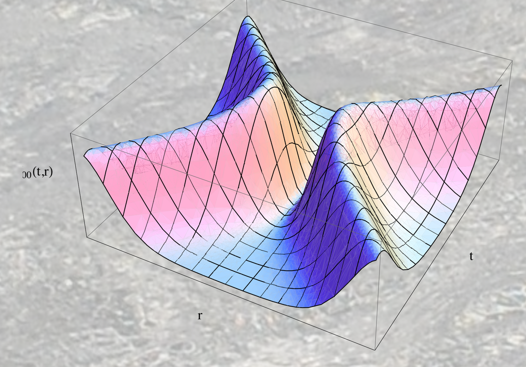

The Fluid–Gravity Correspondence

QMAP: Center for Quantum Mathematics & Physics

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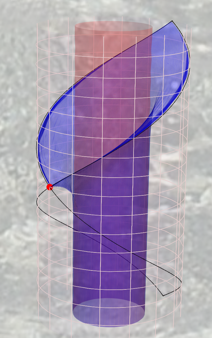
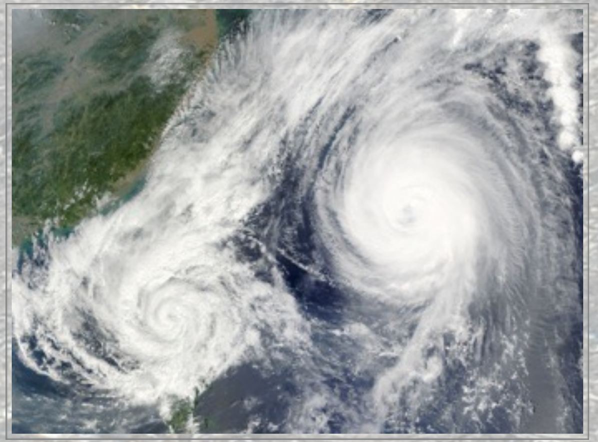


Fluid Dynamics

Fluid dynamics is relevant in the everyday world and describes the collective behaviour of large number of particles in thermal motion.



Macroscopic phenomena like the formation of hurricanes can be modelled by fluid dynamics.

(a)

Fluid dynamics is mathematically described via the famous Navier-Stokes equations. These simply encapsulate the conservation of energy and momentum in the system.

For incompressible, non-relativistic fluids these are:

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \nu\nabla^2\mathbf{v} + \mathbf{f}\,, \qquad \nabla\cdot\mathbf{v} = 0$$

Break up of steady flow by external forcing; this can be the onset of turbulent behaviour.

Relativistic generalizations of these equations are relevant in astrophysics and in particle physics.

Despite a century and half of intense scrutiny, there are many unanswered questions ranging from the physics of *turbulence*, to the mathematical properties of the equations.

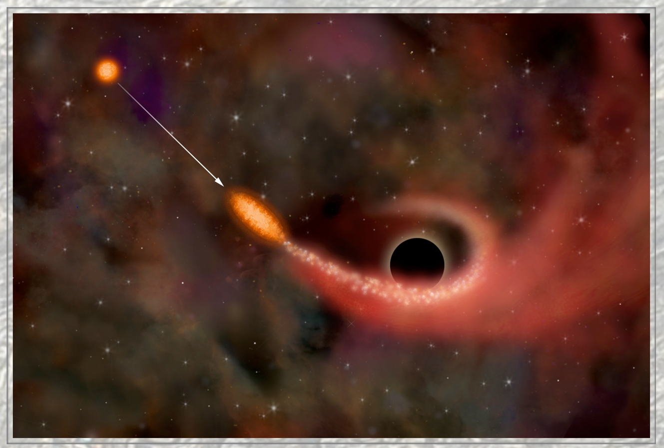
This is recognised by the Clay Mathematical Institute among the seven Millennium problems.

Fully developed turbulent flow occuring when the Reynolds number characterized by the friction coefficient ν becomes large.

(b)

General Relativity

General Relativity is the classical theory of gravity describing the large scale behaviour in the universe. It is relevant for astrophysics and cosmology.

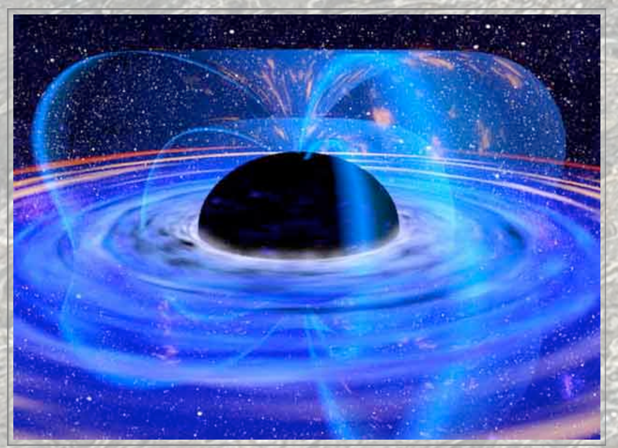


Acceretion of stellar matter by a black hole. Gravitational effects are so strong that they rip stars apart.

(c)

Einstein's equations which describe gravity, imply that the effects of gravitation are produced by the curvature of spacetime.

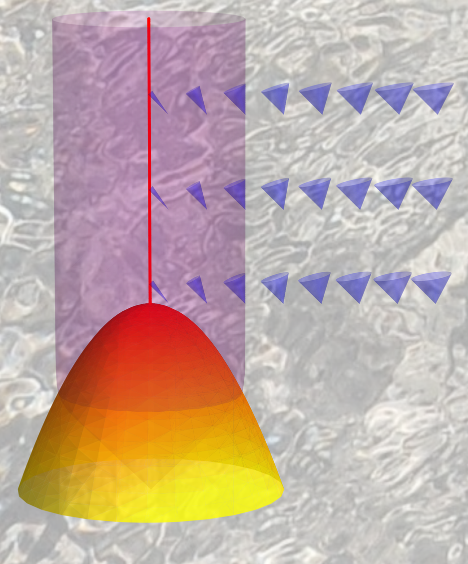
$$R_{\mu\nu} - \frac{1}{2}\, R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = 8\pi\, G_N\, T_{\mu\nu}$$



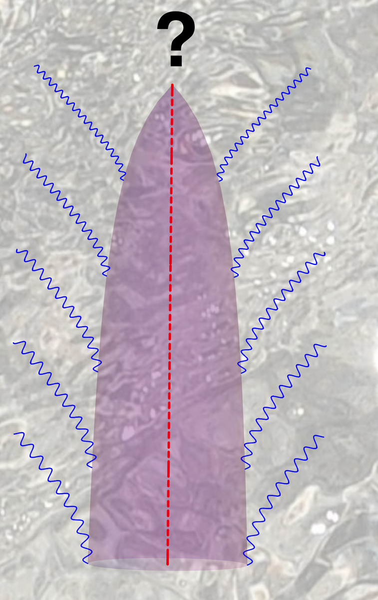
Black holes are classical solutions to Einstein's equations. Due to the gravitational attraction, classically nothing, not even light, escapes.

(d)

Despite decades of research, there are many open issues regarding the nature of general relativity. These range from the famous black hole information paradox, to the nature of spacetime singularities. These questions can be addressed in *string theory* which unifies general relativity and quantum mechanics.



Classically, a star collapses and forms a black hole with a spacetime singularity (red line).



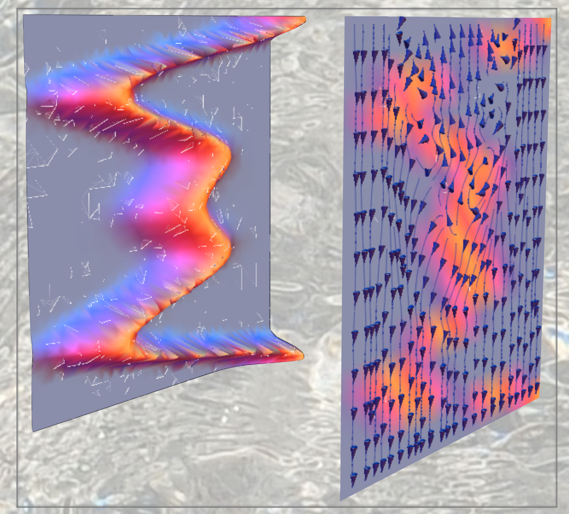
Quantum process cause the black hole to evaporate; the outgoing radiation has no memory of the star, which contradicts rules of quantum mechanics.

The Fluid-Gravity Correspondence

Remarkably, there is an intimate correspondence between the physics of fluids and the equations of general relativity [1,2].

In particular, there is a detailed map that allows us to construct a black hole solution given any fluid configuration. The characteristics of the black hole are completely determined by the behaviour of the fluid.

However, the black hole lives in an auxiliary five dimensional spacetime, whereas the fluid lives in four dimensions.



The fluid hologram (right) captures the entire physics of the higher dimensional spacetime including the black hole (horizon shown on the left).

This correspondence is *holographic*.

The fluid is a hologram capturing accurately the higher dimensional gravitational physics (which is encoded in a non-local fashion).

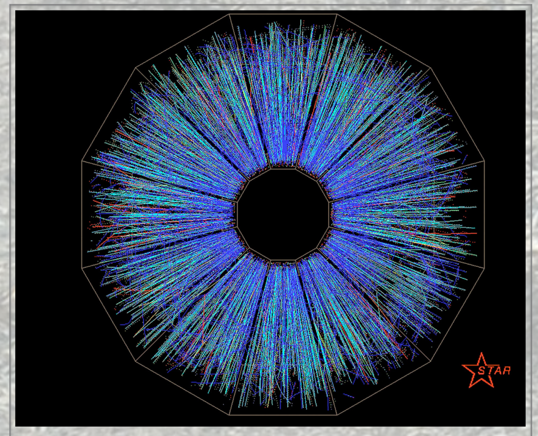
This correspondence naturally arises from a profound connection between Quantum Gravity and quantum field theories (such as those that describe particle physics) called the AdS/CFT correspondence.

As one might imagine, this has far-reaching consequences for some of the deep questions in fluid dynamics and general relativity.

This is the most promising framework to unlock the deep mysteries of Quantum Gravity and the nature of spacetime.

Real-World Applications

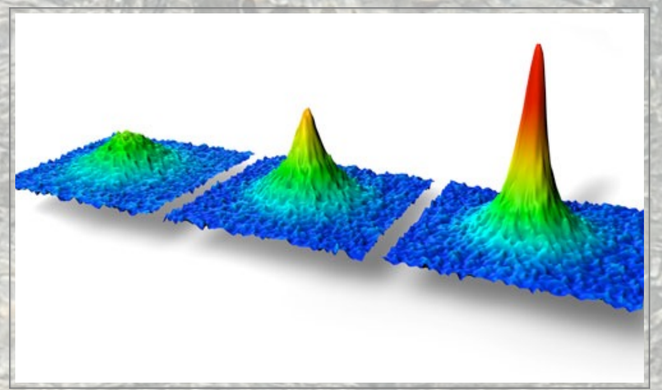
The physics of fluids derived via the fluid-gravity correspondence is playing an important role in understanding the dynamics of the exotic states of matter.



Tracks of the particles produced in heavy ion collisions leading to the quark-gluon plasma, which is about 250,000 times hotter than the center of the sun!

(e)

The hot quark-gluon plasma which simulates the conditions soon after the Big Bang, behaves as a nearly ideal fluid, flowing with very little friction!



Bose-Einstein condensate of fermionic atoms. These systems have led to the discovery of new quantum states of matter.

(f)

At another extreme, ultra-cold Fermi atoms at few nano-Kelvin also behave as ideal fluids.

Properties of such fluids can be derived from the fluid-gravity correspondence [3,4].

There is currently considerable effort to apply these holographic methods to the real world.

References:

[1] *Non-linear fluid dynamics from gravity*, S. Bhattacharyya, V. E. Hubeny, S. Minwalla, and M. Rangamani JHEP **0802**:045, 2008.

[2] *Gravity & Hydrodynamics: Lectures on the fluid-gravity correspondence*, M. Rangamani, CQG **26**, 224003, 2009.

The fluid/gravity correspondence, V. Hubeny, S. Minwalla & M. Rangamani, in Black Holes in Higher Dimensions (ed. G.Horowitz) CUP 2012.

[3] *Viscosity in strongly interacting quantum field theories from black hole physics*, P. Kovtun, D.T. Son, and A. Starinets, PRL 94, 1995.

[4] *Heating up Galilean holography, C.P.Herzog, M. Rangamani, S.F. Ross, JHEP 0811:080 2008.*

Images courtesy:
a) Hurricanes Parma & Mellor, NASA.
b) Wake vortex turbulence, Wikimedia Commons.
c) Black hole accretion, M. Weiss, CXC, NASA.
d) Artist's conception of a black hole, XMM-Newton (ESA).
e) Relativistic Heavy Ion Collider, STAR detector.
f) Fermionic BEC, Markus Griener, JILA, USA.