4-Gluon Amplitude at 3-loops in QCD

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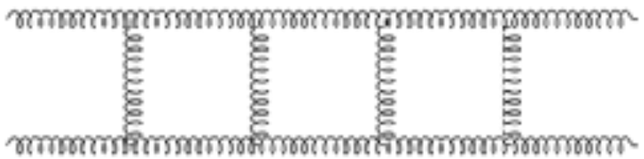
PROLOGUE

- Scattering amplitude is the basic building block in QFT
- More precise theoretical predictions are crying need at the LHC
- Necessary to reveal the underlying structures in QFT
- We focus on a 4-gluon process involving on-shell partons in QCD

$$g(p_1) + g(p_2) + g(p_3) + g(p_4) \to 0$$

- Ingredient for the di-jet production
- State-of-the-art
  - 1-loop: Ellis, Sexton '86
  - 2-loop Helicity amplitude: Bern, Dixon, Kosower '00
  - 2-loop full amplitude: Glover, Oleari, Yeomans '01
- Next challenging goal: go for 3-loop!
- First attempt in N=4 by Henn, Mistlberger in '16
- We address this 4-point amplitude at 3-loop for the first time in QCD

Our goal



Process of Interest

- We consider

$$g(p_1) + g(p_2) + g(p_3) + g(p_4) \to 0 \qquad \text{on-shell in QCD}$$

with arbitrary number of light fermions in fund represent of SU(N) gauge group

- Goal: Compute the amplitude and helicity amplitudes in planar limit

Include all the terms satisfying $n_f^{a_1} N^{a_2} a_s^{a_3}$ with $a_1 + a_2 = a_3$

- One approach: decompose the amplitude into linearly independent tensor structures
- 138 Tensorial Structures

constraints
- Transversality Condition $\epsilon_i . p_i = 0$
- Gauge Fixed Tensor $\epsilon_1 . p_2 = \epsilon_2 . p3 = \epsilon_3 . p_4 = \epsilon_4 . p_1 = 0$

Number of tensorial structures reduce to 10

- All plus amplitude

$$|\mathcal{M}_{++++}\rangle = \sum_{i=1}^{10} A_i T_{i,++++} = \frac{1}{4\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}\Big\{ -4(A_1 + A_3)st - 4A_2 u^2 - 2A_4 stu + 2A_5 su^2 - 2A_6 stu$$

$$-2A_7 stu + 2A_8 tu^2 - 2A_9 stu - A_{10} stu^2 \Big\}$$

Computation

- Age-old Feynman diagrammatic approach, however, lots of challenges!
- Huge expressions, complicated reductions!
- Feynman diagrams using `Qgraf`: 48723 three-loop, 39k planar topologies — Noguira '06
- Discard non-planar diagrams by removing sub-leading colors.
- Cross-checked using `REDUZE2` using the liberty of shifting loop momenta — Monteuffel, Studerus '12
- In-house `FORM` code: convert `Qgraf` raw output to `FORM`

  SU(N) color simplification

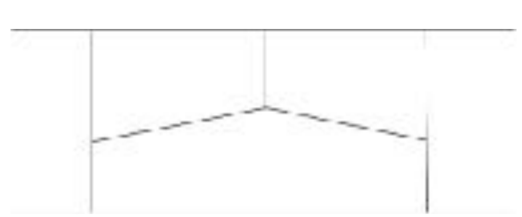
  Lorentz algebra } in d-spacetime dimensions — Vermaseren

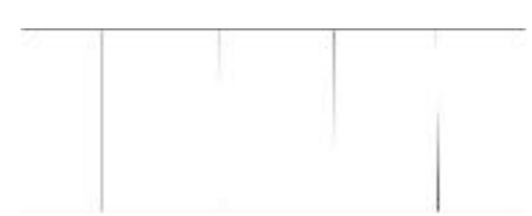
  Dirac algebra }
- Millions of 3-loop integrals with very high power of numerators: highest 6!

$$J[a_1, a_2, \cdots, a_{15}] = \int \prod_{L=1,2,3} \left[\frac{d^d k_L}{(2\pi)^d}\right] \frac{1}{D_1^{a_1} D_2^{a_2} \cdots D_{15}^{a_{15}}}$$

D are the inverse of propagators

Most complicated $\sum |a_i| = 16$





COMPUTATION

- IBP Reduction

  FIRE5.2 C++ along with LiteRed1.82

  2-step reductions:

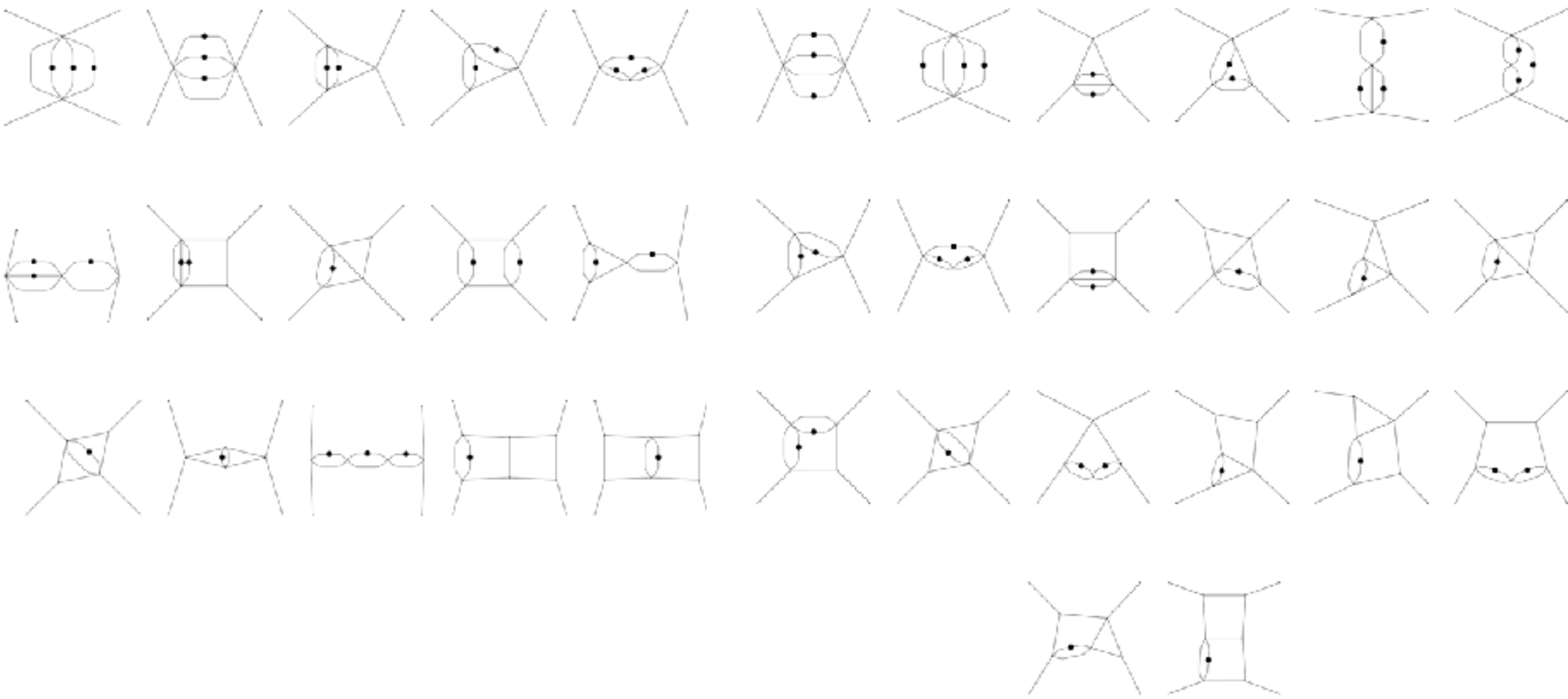
  1. LiteRed along with Mint: symbolic rules & 89 MIs
  2. Non-minimal set : Huge reduction file!
  3. Reduce again using FIRE c++: 81 MIs — Boels, Jin, Luo ‘18
  4. Minimal set: reduction file size reduced 1/10

  Advantage: substantial improvement of total reduction time

  Comments: Non-minimal set of MIs is inefficient

- 21 MIs with one double prop
- 2 MIs with one numerator

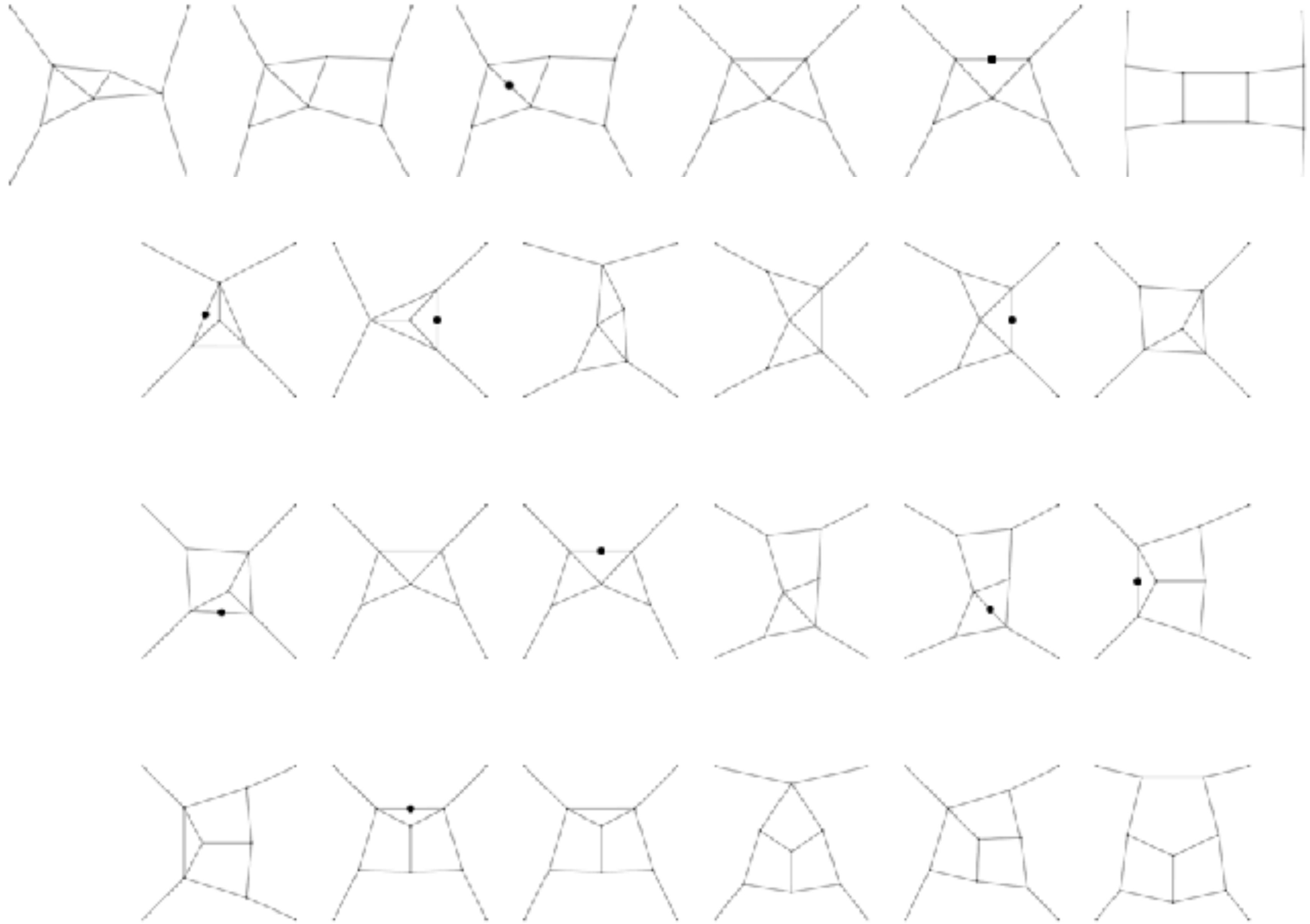
Henn, Smirnov, Smirnov ’13



With Bubble Sub-diagrams

MIs in UT-Basis

Henn, Smirnov, Smirnov '13



MIs without Bubble sub-integrals

- Remaining integrals are obtained by interchanging s & t

UV & IR

- Dimensional regularisation: $d = 4 - 2\epsilon$
- UV structures of amplitude and all plus amplitude are different

$\mathcal{M}^{(0)}(s,t,\epsilon) \neq 0$ UV divergent 1-loop

$\mathcal{M}^{(0)}_{++++}(s,t,\epsilon) = 0$ All plus amplitude vanishes at tree level → UV Finite 1-loop

UV renormalisation in MS-bar

- The IR poles are universal and were first predicted by Catani up to 2-loop (except single pole)

Catani '98

Sterman, Yeomans '03

- From SCET

$$\mathcal{M}^{fin}(s,t) = \lim_{\epsilon \to 0} \mathbf{Z}^{-1}(\epsilon,s,t)\mathcal{M}(\epsilon,s,t)$$

UV finite

All the IR divergences are governed by the matrix $\mathbf{Z}$

The all order solution:

$$\mathbf{Z}(\epsilon,s,t,\mu) = \mathcal{P}\exp\left[\int_{\mu}^{\infty} \frac{d\mu'}{\mu'}\mathbf{\Gamma}(s,t,\mu')\right]$$

Becher & Neubert '09

Gardi & Magnea '09

Results & Conclusions

- Our 1 and 2-loop results agree with the universal IR structure and the literatures.
- We have checked the amplitude as well as ++++ amplitude
- They exhibit the necessary symmetry between s & t
- We have extended the 2-loop results to higher order in ϵ
- 3-loop reduction is done!
- Result in terms of Master integrals is at hand.
- Just waiting for the final result! Will be available soon!
- First step towards the full computation
- First ever attempt in QCD!

Thank you!

Preliminary Result (Partial of course)

Coefficient of. $n_f^3 \; Tr(T^{a_1}T^{a_3}T^{a_4}T^{a_2})$ of the Amplitude (Unrenormalised)

$$\frac{1}{\epsilon}\frac{1}{(1+x)^3}\Bigg[T_1\left(\frac{8x^4}{27}+\frac{28x^3}{27}+\frac{4x^2}{3}+\frac{20x}{27}+\frac{4}{27}\right)+T_2\left(-\frac{4x^3}{27}-\frac{4x^2}{27}+\frac{4x}{27}+\frac{4}{27}\right)+T_5\left(\frac{8}{27t}-\frac{8x^2}{27t}\right)$$
$$+T_8\left(\frac{8}{27t}-\frac{8x^2}{27t}\right)+T_{10}\left(\frac{16}{27t^2}-\frac{16x}{27t^2}\right)\Bigg]$$

$$+\frac{1}{(1+x)^3}\Bigg[T_1\left\{\left(-\frac{8x^4}{9}-\frac{28x^3}{9}-4x^2-\frac{20x}{9}-\frac{4}{9}\right)\Big(H(\{0\},x)+\log(-s)-3\Big)\right\}$$
$$+T_2\left\{\left(\frac{4x^3}{9}+\frac{4x^2}{9}-\frac{4x}{9}-\frac{4}{9}\right)\Big(\log(-s)-3\Big)\right\}$$
$$+T_5\left\{\left(\frac{8x^2}{9t}-\frac{8}{9t}\right)\Big(\log(-s)-3\Big)\right\}+T_8\left\{\left(\frac{8x^2}{9t}-\frac{8}{9t}\right)\Big(\log(-s)-3\Big)\right\}$$
$$+T_{10}\left\{\left(\frac{16x}{9t^2}-\frac{16}{9t^2}\right)\Big(\log(-s)-3\Big)\right\}\Bigg]$$

Up to an overall normalisation factor $\qquad x=\frac{u}{t}\,,\, t=2p_1.p_3,\, u=2p_2.p_3$

Absolutely no check has been performed! Will be done soon!

Large N Limit

- Amplitude is a tensor in Color space
- Can be decomposed in terms of traces of fundamental color generators of SU(N)
- Pure gauge amplitude can be expressed in terms of six color structures

$$C_1 = tr(1234) + tr(1432) \qquad C_4 = tr(12)tr(34)$$
$$C_2 = tr(1243) + tr(1342) \qquad C_5 = tr(13)tr(24)$$
$$C_3 = tr(1423) + tr(1324) \qquad C_6 = tr(14)tr(23)$$

$$tr(T^{a_1}T^{a_2}T^{a_3}T^{a_4}) = tr(1234)$$

It can be written as

$$A_i^{(L)} = \sum_{\lambda=1}^{3}\left\{\sum_{k=0}^{\lfloor \frac{L}{2} \rfloor} N^{L-2k} A_{i,\lambda}^{(L),2k}\right\} C_\lambda + \sum_{\lambda=4}^{6}\left\{\sum_{k=0}^{\lfloor \frac{L-1}{2} \rfloor} N^{L-2k-1} A_{i,\lambda}^{(L),2k+1}\right\} C_\lambda$$

$A_{i,\lambda}^{(L),0}$ are leading order in N. Others are sub-leading

Only $A_{i,\lambda}^{(L),0}$ contributes in the large N-limit

Our goal

Only planar diagrams contribute: Planar limit