Benchmarking planar five-parton two-loop QCD amplitudes with numerical unitarity

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Motivation

Precision era at the LHC

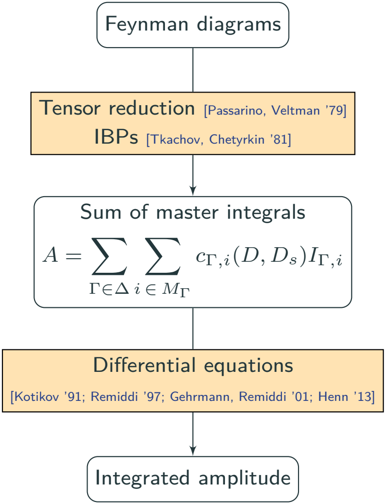
- No direct detection of new physics $\implies$ zoom in into data
- High order calculations, i.e. NNLO are required to achieve $\approx 1\%$ level accuracy theory predictions for signal and background

State of the art

- Most of $2 \to 2$ processes are available at NNLO, but many interesting processes have > 2 particles in the final state
- Handling IR divergences for > 2 particles is very challenging, active research by many groups
- Huge effort towards computation of multi-scale Feynman integrals
- $2 \to 3$ two-loop amplitude frontier is being actively attacked and first simplest amplitudes have been benchmarked

We focus on integrand reduction of two-loop amplitudes with numerical unitarity method.

The Standard Approach to General Two-loop Amplitudes



Challenges:

- Large intermediate expressions
- Generating IBP relations is practically difficult

Two-loop numerical unitarity tries to avoid these issues:

- Only a restricted set of IBP relations is required for each topology
- Implicit numerical reduction to master integrals
- Full numerical framework avoids expression bloat

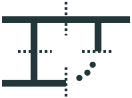
Two-Loop Reduction to Masters with Numerical Unitarity

1. Take an ansatz for loop-amplitude integrand, decomposing into master (M_Γ) and surface (S_Γ) integrands [Ita '15].

$$\mathcal{A}(\ell_l) = \sum_{\text{Topologies } \Gamma} \sum_{i \in M_\Gamma \cup S_\Gamma} \frac{c_{\Gamma,i}\, m_{\Gamma,i}(\ell_l)}{\prod_{\text{props } j} \rho_j}.$$

2. For each topology build linear systems (cut equations) for master/surface coefficients $c_{\Gamma,i}$ by putting loop momenta on-shell.

$$\sum_{i \in M_\Gamma \cup S_\Gamma} c_{\Gamma,i}(D, D_s)\, m_{\Gamma,i}(\ell_l^\Gamma) =$$



$$- \sum_{\text{ancestors } \Gamma'} \frac{N(\Gamma', \ell_l^\Gamma)}{\prod_{k \in P_{\Gamma'} \setminus P_\Gamma} \rho_k(\ell_l^\Gamma)}$$

3. Invert linear systems (e.g. by PLU or QR factorization) for given kinematics, D and D_s
4. Reconstruct rational functions of D and D_s by sampling $\Rightarrow$ master coefficients directly from on-shell data.
5. Combine with master integrals $\Rightarrow$ integrated amplitude

Status

The BH2 Project

We are constructing a C++ framework for D-dimensional multi-loop numerical unitarity. We implement algorithms suitable for multi-precision floating point as well as exact arithmetics (finite fields $\rightarrow$ rational numbers).

Collaboration

Samuel Abreu, Jerry Dormans, Fernando Febres-Cordero, Harald Ita, Matthieu Jaquier, Ben Page, Evgenij Pascual, VS

Results so far

- 4 point Yang-Mills amplitudes [arXiv:1703.05273]: reproduced analytic results from literature [Bern, De Freitas, Dixon '02]
- benchmark 5 point Yang-Mills amplitudes [arXiv:1712.03946] (see also [Badger et al., arXiv:1712.02229])
- Reproduced known N_f-contributions to 4-gluon amplitudes

Outlook

What's next?

- Extension to full QCD spectrum and beyond. Challenges:
  - dim. reg with fermions in numerical framework [arXiv:1803.11127]
  - no square roots (of scalar products) allowed for exact arithmetics (as in $\ell_{[D]}$)
  - efficient colour decomposition with quarks
- Functional reconstruction of full kinematical dependence of integral coefficients
- Numerical stability and performance improvements $\Rightarrow$ *integrated* virtual matrix elements
- Non-planar topologies: (multiple) non-coloured particles in the final state; sub-leading colour contributions
- Long term goal: combine with other bits of NNLO computation to deliver full NNLO precise predictions for multi-scale processes

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Stay tuned!