BMS Supertranslation Symmetry Implies Faddeev-Kulish Amplitude

Sangmin Choi

June 12, 2018

Amplitudes Summer School 2018

Based on...

"BMS supertranslation symmetry implies Faddeev-Kulish amplitude"

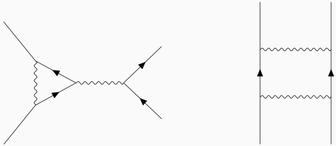
JHEP 1802 (2018) 171, arXiv:1712.04551

Sangmin Choi, Ratindranath Akhoury

Infrared divergence

Consider a scattering amplitude $\langle q_1, q_2|\mathcal{S}|p_1, p_2\rangle$ in QED.

Loops diagrams have infrared divergences.



These divergences exponentiate, and the amplitude vanishes in the limit where the infrared regulator is removed:

$$\langle q_1, q_2|\mathcal{S}|p_1, p_2\rangle = 0$$

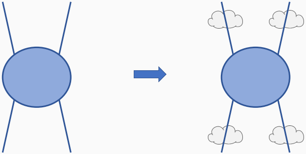
Traditionally, this problem has been circumvented at the level of *cross section* via the Bloch-Nordsieck method; the *S-matrix elements* are left ill-defined.

Faddeev-Kulish states

An alternative approach is to replace Fock states with dressed (Faddeev-Kulish, FK) states:

$$|\mathbf{p}\rangle \quad \rightarrow \quad e^{R(\mathbf{p})}\,|\mathbf{p}\rangle\,,$$

where $R(\mathbf{p})$ is an anti-Hermitian operator containing soft gauge particles.



$\langle$out$|\mathcal{S}|$in$\rangle$ $\langle$out$|e^{-R}\mathcal{S}e^{R}|$in$\rangle$

Amplitudes built using FK states (FK amplitudes) are free of infrared divergences.

Large gauge symmetry

Gauge/gravity theories have asymptotic symmetries:

- Large gauge symmetry for QED.
- BMS symmetry for gravity.

Charges of asymptotic symmetries should be conserved:

$$\langle \text{out}| \, [Q, \mathcal{S}] \, |\text{in}\rangle = 0.$$

However, Fock states do not conserve this charge. Infrared divergence reflects this violation of charge conservation. [Kapec, Perry, Raclariu, Strominger '17]

The FK states are charge eigenstates of the BMS supertranslation:

$$Q e^{R(\mathbf{p})} |\mathbf{p}\rangle = \underbrace{C(\mathbf{p})}_{\propto p} e^{R(\mathbf{p})} |\mathbf{p}\rangle \,.$$

Charge conservation follows from energy-momentum conservation.

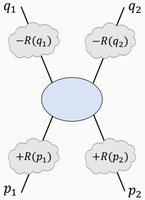
But there are more BMS eigenstates then there are FK states. For example,

$$e^{R(\mathbf{q})} |\mathbf{p}\rangle \,, \qquad \mathbf{q} \neq \mathbf{p},$$

is also a BMS eigenstate but not an FK state.

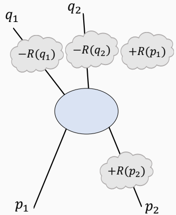
BMS Supertranslation Charge

An FK amplitude looks like

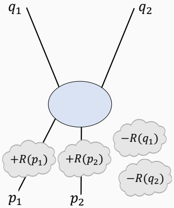


$$= \langle \mathbf{q}_1, \mathbf{q}_2 | \, e^{-R(\mathbf{q}_1)-R(\mathbf{q}_2)} \; \mathcal{S} \; e^{R(\mathbf{p}_1)+R(\mathbf{p}_2)} \, | \mathbf{p}_1, \mathbf{p}_2 \rangle \, .$$

The following amplitudes also conserve BMS charge:



$$\langle f | e^{-R(\mathbf{q}_1)-R(\mathbf{q}_2)+R(\mathbf{p}_1)} \mathcal{S} e^{R(\mathbf{p}_2)} | i \rangle$$



$$\langle f | \mathcal{S} e^{R(\mathbf{p}_1)+R(\mathbf{p}_2)-R(\mathbf{q}_1)-R(\mathbf{q}_2)} | i \rangle$$

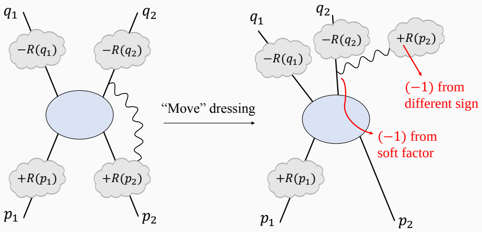
But an FK amplitude is infrared-finite. Are these also infrared-finite? [Kapec, Perry, Raclariu, Strominger '17]

Infrared-finiteness

From the formula for the leading term of loop diagrams [Choi, Kol, Akhoury '17],

$$(-1)^N \left[\prod_{r=1}^{N+N'} \int \frac{d^3 k_r}{(2\pi)^3 (2\omega_r)} f_{\mu\nu} I^{\mu\nu,\rho_r\sigma_r} \right] \mathcal{J}_{\rho_1\sigma_1\cdots\rho_{N+N'}\sigma_{N+N'}}$$

the net effect of "moving" a dressing from the in-state to the out-state can be summarized in the following diagram:



$\Rightarrow$ no net effect on the leading term of the amplitude.

Since FK amplitude is infrared-finite, all amplitudes that conserve BMS supertranslation charge are infrared-finite.

Summary

To summarize the main points:

- Conventional S-matrix elements vanish due to infrared divergences. This is a penalty for violating charge conservation of the asymptotic symmetries.
- FK amplitudes are well defined – i.e. they do not exhibit infrared divergence.
- There thus is a close connection between asymptotic symmetries and FK states: The set of FK states is a subset of charge eigenstates that automatically conserve the charge of asymptotic symmetry.
- However, any amplitude that conserves the charge (and therefore is non-zero) is equivalent to the corresponding FK amplitude at the leading order.