6D dual superconformal algebra

Michal Pazderka

Masaryk University, Czech Republic, Brno

M. Pazderka, K. Bering
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Motivation

- Possible symmetries of massive 4D $\mathcal{N} = 4$ SYM?
- Relation between symmetries of $\mathcal{N} = 4$ SYM and $\mathcal{N} = 6$ ABJM?

How?

- The generators must commute with constraints

$$C_1 := x_i^{EF} - x_{i+1}^{EF} - \lambda_i^{Ea}\lambda_{ia}^{F} \approx 0, \qquad C_2 := \theta_i^{IA} - \theta_{i+1}^{IA} - \eta_i^{Ia}\lambda_{ia}^{A} \approx 0$$

- The generators form a superconformal algebra.

- Ansatz for dual generators (in analogy with 4D)

$$P_{AB} = \sum_{i=1}^{n+1} \frac{\partial}{\partial x_i^{AB}}, \qquad Q_{IA} = \sum_{i=1}^{n+1} \frac{\partial}{\partial \theta_i^{IA}}$$

Result

- We found all generators in analogy with $4D\ \mathcal{N}=4$ SYM (**and MORE!**):

$$\{P, M, D, K, Q, \bar{Q}, S, \bar{S}, R, C\}$$

- Examples:

$$\bar{Q}^J_B = \sum_i \left\{ \theta_i^{JC} \partial_{iBC} + \frac{1}{2}\eta_i^{Ja}\partial_{iBa} - \frac{1}{2} y_i^{JK}\partial_{iKB} \right\}$$

$$K^{AD} = 2\alpha \sum_i \left\{ x_i^{[AE}\theta_i^{MD]}\frac{\partial}{\partial\theta_i^{ME}} - x_i^{[AB}x_i^{CD]}\frac{\partial}{\partial x_i^{BC}} + \theta_i^{M[A}\theta_i^{NB]}\frac{\partial}{\partial y_i^{MN}} \right.$$
$$\left. + \frac{1}{2}\left(x_i^{[AE} + x_{i+1}^{[AE}\right)\lambda_i^{D]a}\frac{\partial}{\partial\lambda_i^{Ea}} + \frac{1}{2}\lambda_i^{[Aa}\left(\theta_i^{MD]} + \theta_{i+1}^{MD]}\right)\frac{\partial}{\partial\eta_i^{Ma}} \right\}$$

Result

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- Examples:

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$$\begin{aligned} K^{AD} =& 2\alpha \sum_i \left\{ x_i^{[AE} \theta_i^{MD]} \frac{\partial}{\partial \theta_i^{ME}} - x_i^{[AB} x_i^{CD]} \frac{\partial}{\partial x_i^{BC}} + \theta_i^{M[A} \theta_i^{NB]} \frac{\partial}{\partial y_i^{MN}} \right. \\ & \left. + \frac{1}{2} \left(x_i^{[AE} + x_{i+1}^{[AE} \right) \lambda_i^{D]a} \frac{\partial}{\partial \lambda_i^{Ea}} + \frac{1}{2} \lambda_i^{[Aa} \left(\theta_i^{MD]} + \theta_{i+1}^{MD]} \right) \frac{\partial}{\partial \eta_i^{Ma}} \right\} \end{aligned}$$

Superization

- New variables:

$$\Lambda_i^{\mathcal{A}a} = \begin{pmatrix} \lambda_i^{Aa} \\ \eta_i^{Ia} \end{pmatrix}, \qquad X_i^{\mathcal{AB}} = \begin{pmatrix} x_i^{AB} & \theta_i^{AJ} \\ \theta_i^{IB} & y_i^{IJ} \end{pmatrix}$$

- Super-constraints:

$$X_i^{\mathcal{AB}} - X_{i+1}^{\mathcal{AB}} - \Lambda_i^{\mathcal{A}a} \Lambda_{ia}^{\mathcal{B}} \approx 0$$

Generators

$$\mathbb{P}_{\mathcal{A}\mathcal{B}} = \sum_i \frac{\partial}{\partial X_i^{\mathcal{A}\mathcal{B}}}$$

$$\mathbb{M}^{\mathcal{A}}{}_{\mathcal{B}} = \sum_i \left\{ X_i^{\mathcal{A}\mathcal{C}} \frac{\partial}{\partial X_i^{\mathcal{B}\mathcal{C}}} + \frac{1}{2} \Lambda^{\mathcal{A}a} \frac{\partial}{\partial \Lambda_i^{\mathcal{B}a}} \right\}$$

$$\mathbb{D} = -\sum_i \left\{ X_i^{\mathcal{A}\mathcal{B}} \frac{\partial}{\partial X_i^{\mathcal{A}\mathcal{B}}} + \frac{1}{2} \Lambda^{\mathcal{A}a} \frac{\partial}{\partial \Lambda_i^{\mathcal{A}a}} \right\}$$

$$\begin{aligned}\mathbb{K}^{\mathcal{A}\mathcal{B}} = \alpha_S \sum_i \Bigg\{ & (-1)^{|\mathcal{B}|(|\mathcal{C}|+|\mathcal{D}|)} X_i^{\mathcal{A}\mathcal{C}} X_i^{\mathcal{D}\mathcal{B}} \frac{\partial}{\partial X_i^{\mathcal{C}\mathcal{D}}} \\ & + \frac{1}{2}(-1)^{|\mathcal{B}||\mathcal{C}|+1} \left(X_i^{\mathcal{A}\mathcal{C}} + X_{i+1}^{\mathcal{A}\mathcal{C}} \right) \Lambda_i^{\mathcal{B}a} \frac{\partial}{\partial \Lambda^{\mathcal{C}a}{}_i} \Bigg\} \\ & + (-1)^{|\mathcal{A}||\mathcal{B}|+1} (\mathcal{A} \leftrightarrow \mathcal{B})\end{aligned}$$

Algebra

$$[\mathbb{P}_{\mathcal{AB}}, \mathbb{D}] = -\mathbb{P}_{\mathcal{AB}} \ , \qquad \left[\mathbb{P}_{\mathcal{CD}}, \mathbb{M}^{\mathcal{A}}{}_{\mathcal{B}}\right] = \delta^{\mathcal{A}}_{\mathcal{C}} \mathbb{P}_{BD} + (-1)^{|\mathcal{D}||\mathcal{C}|+1} \delta^{\mathcal{A}}_{\mathcal{D}} \mathbb{P}_{\mathcal{BC}}$$

$$\left[\mathbb{D}, \mathbb{K}^{\mathcal{AB}}\right] = -\mathbb{K}^{\mathcal{AB}} \ , \qquad \left[\mathbb{M}^{\mathcal{A}}{}_{\mathcal{B}}, \mathbb{K}^{\mathcal{EF}}\right] = \delta^{\mathcal{E}}_{\mathcal{B}} \mathbb{K}^{\mathcal{AF}} + (-1)^{|\mathcal{E}||\mathcal{F}|+1} \delta^{\mathcal{F}}_{\mathcal{B}} \mathbb{K}^{\mathcal{AE}}$$

$$\left[\mathbb{M}^{\mathcal{A}}{}_{\mathcal{B}}, \mathbb{M}^{\mathcal{E}}{}_{\mathcal{F}}\right] = \frac{1}{2} \delta^{\mathcal{E}}_{\mathcal{B}} \mathbb{M}^{\mathcal{A}}{}_{\mathcal{F}} + (-1)^{(|\mathcal{A}|+|\mathcal{B}|)(|\mathcal{E}|+|\mathcal{F}|)+1} \delta^{\mathcal{A}}_{\mathcal{F}} \mathbb{M}^{\mathcal{E}}{}_{\mathcal{B}}$$

$$\begin{aligned} \left[\mathbb{K}^{\mathcal{AB}}, \mathbb{P}_{\mathcal{EF}}\right] =& \alpha_S \left[(-1)^{|\mathcal{B}||\mathcal{F}|+1} \delta^{\mathcal{A}}_{\mathcal{E}} \mathbb{M}^{\mathcal{B}}{}_{\mathcal{F}} + (-1)^{|\mathcal{E}||\mathcal{F}|+1} (\mathcal{E} \leftrightarrow \mathcal{F})\right] \\ & + (-1)^{|\mathcal{A}||\mathcal{B}|+1} (\mathcal{A} \leftrightarrow \mathcal{B}) \end{aligned}$$

Thank you for your attention!