SU(N) characters and thermal statistics in a string bit model[1][2]

Sourav Raha

Department of Physics, University of Florida

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[1] S. Raha, Phys. Rev. D 96, 086006 (2017)

[2] T.L. Curtright, S. Raha and C.B. Thorn, Phys. Rev. D 96, 086021 (2017)

What are string bits?

Charles Thorn's attempt at a nonperturbative formulation of string theory.

String bits are units of lightcone-quantized strings.
Each bit carries an infinitesimal unit of P^+ of the emergent string.
No longitudinal space, x^- emerges when σ becomes continuous.

Creation operators are $N \times N$ matrices in "color"[3].

With Oren Bergman[4] formulation extended to include superstrings.

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[3] C.B. Thorn, Proceedings of Sakharov Conf. on Physics, Moscow (1991) 447-454, hep- th/9405069.

[4] O. Bergman and C.B. Thorn, Phys. Rev. D 52, 5980 (1995)

The simplest string bit model[5]

$$\bar{\Phi}{}^{\alpha}_{\beta}(\theta) = \bar{a}{}^{\alpha}_{\beta} + \bar{b}{}^{\alpha}_{\beta}\theta \qquad \alpha, \beta \in 1 \ldots N$$

$$a{}^{i}_{j} = \left(\bar{a}^{\dagger}\right)^{j}_{i} \qquad b{}^{i}_{j} = \left(\bar{b}^{\dagger}\right)^{j}_{i}$$

$$\left[a{}^{i}_{j}, \bar{a}{}^{k}_{l}\right] = \delta^{i}_{l}\delta^{k}_{j} \qquad \left\{b{}^{i}_{j}, \bar{b}{}^{k}_{l}\right\} = \delta^{i}_{l}\delta^{k}_{j}$$

Some examples:

Closed chain: $\mathrm{tr}\left[\bar{a}\bar{b}\bar{a}\bar{a}\right]|0\rangle$, $\mathrm{tr}\left[\bar{b}\bar{a}\bar{a}\right]|0\rangle$, etc.

Open chain: $\bar{a}{}^{i}_{j}|0\rangle$, $\bar{b}{}^{i}_{j}|0\rangle$, $\bar{a}{}^{i}_{r}\bar{a}{}^{r}_{s}\bar{a}{}^{s}_{t}\bar{b}{}^{t}_{j}|0\rangle$, $\bar{b}{}^{i}_{r}\bar{a}{}^{r}_{s}\bar{a}{}^{s}_{j}|0\rangle$, etc.

[5] S. Sun and C.B. Thorn, Phys. Rev. D 89, 105002 (2014)

Dynamics

$$\mathcal{P}^0 = \frac{1}{\sqrt{2}}\Big\{ m\mathrm{tr}\left[\bar{a}a + \bar{b}b\right] + \frac{2T_0}{m\pi}\mathrm{tr}\left[\bar{a}a + \bar{b}b\right] +$$
$$\frac{T_0}{2mN}\mathrm{tr}\left[\left(\bar{a}^2 - i\bar{b}^2\right)a^2 - \left(\bar{b}^2 - i\bar{a}^2\right)b^2 + \left(\bar{a}\bar{b} + \bar{b}\bar{a}\right)ba + \left(\bar{a}\bar{b} - \bar{b}\bar{a}\right)ab\right]\Big\}$$

Produces color confinement, open chain states are suppressed.

$$Z \approx \mathrm{Tr}\left[e^{-\beta\mathcal{P}^0}\right]_{\text{singlets}} \to \mathrm{Tr}\left[e^{-\beta\frac{m}{\sqrt{2}}\mathrm{tr}\left[\bar{a}a+\bar{b}b\right]+\mathcal{O}\left(\frac{T_0}{m}\right)}\right] = \mathrm{Tr}\left[x^{\mathcal{M}}\right]$$

Use $SU\left(N\right)$ characters to project out the singlet subspace.

SU(N) characters

Assign a color character[6] to each creation operator. eg.

$$\bar{a}_l^k|0\rangle \quad \xrightarrow{\chi} \quad e^{i(\theta_k-\theta_l)}$$

Characters of irreps satisfy orthonormal relations:

$$\frac{\int_G dg\, \bar{\chi}_R(g)\, \chi_{R'}(g)}{\int_G dg} \quad = \quad \delta_{R,R'}$$

$$Z \quad \propto \quad \int_0^{2\pi} \left(\prod_r d\theta_r\right) \left(\prod_{p<q} \left|e^{i\theta_p} - e^{i\theta_q}\right|^2\right) \frac{1-x}{1+x} \left\{\prod_{k,l} \frac{1+xe^{i(\theta_k-\theta_l)}}{1-xe^{i(\theta_k-\theta_l)}}\right\}$$

[6] T.L. Curtright and C.B. Thorn, Nuclear Physics B 274 (1986).

Results[7]

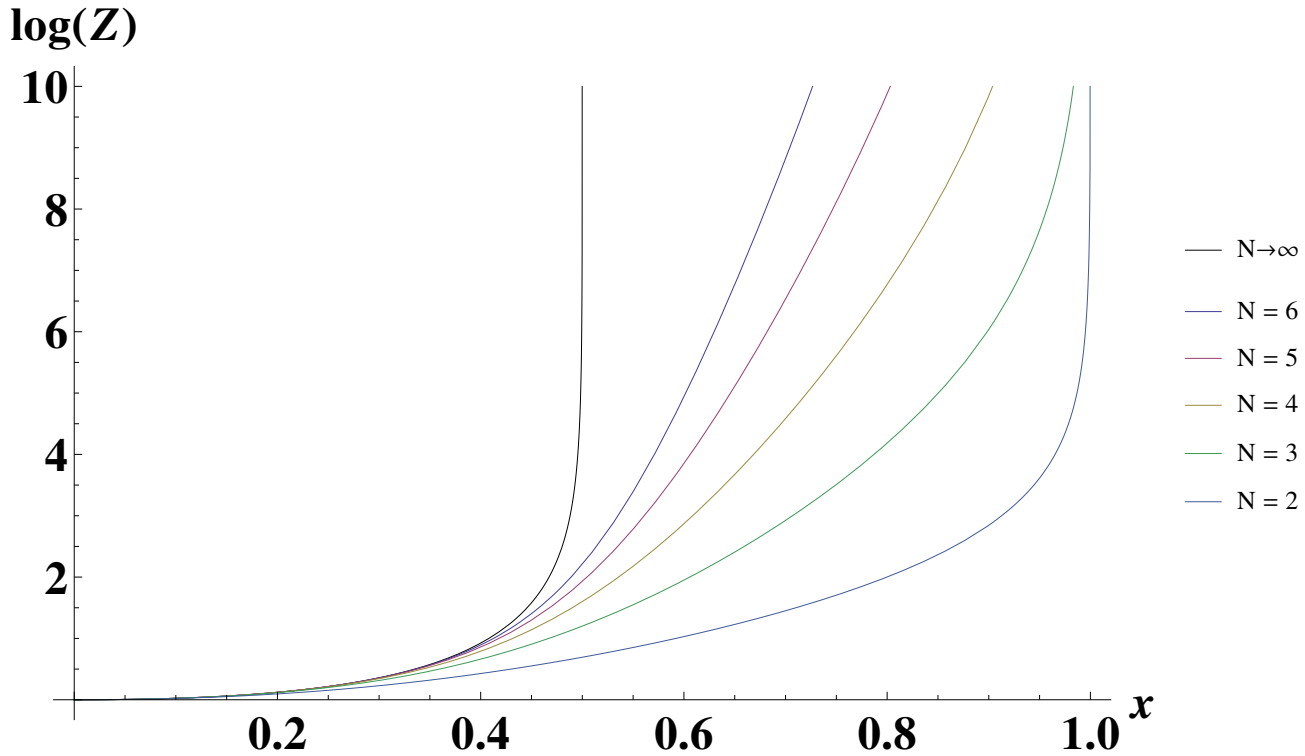

Figure: Partition functions for the singlet sector of the simplest string bit model for different N's. The Hagedorn transition takes place in the large N limit at $x_H = 1/2$.

[7] S. Raha, Phys. Rev. D 96, 086006 (2017)

Thank you!