Amplitudes in $\mathcal{N} = 2$ Chern-Simons Matter Theories

Pranjal Nayak

UNIVERSITY OF KENTUCKY

Amplitudes 2018 Summer School

June 12, 2018

Based on

- arXiv: 1710.04227
  K.Inbasekar, S.Jain, **PN**, V.Umesh
- arXiv: 1711.02672
  K.Inbasekar, S.Jain, S. Majumdar, **PN**, T. Neogi, T. Sharma, R. Sinha, V.Umesh
- ongoing work...
  K.Inbasekar, S.Jain, **PN**, T. Sharma, V.Umesh

Dualities in Chern-Simons theories

Chern-Simons theories coupled to matter have been conjectured to enjoy a strong-weak duality [Giombi, Minwalla, Prakash et. al '11; Aharony, Gur-Ari, Yacoby '12]:

- Free fermions coupled to Chern-Simons are dual to critical bosons coupled to Chern-Simons.
- $\mathcal{N} = 2$ Chern-Simons theory coupled to fundamental matter is self dual

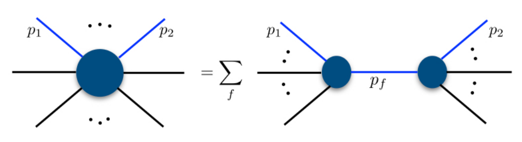
$$\mathcal{S}_{\mathcal{N}=2} = \int d^3x \Big[-\frac{\kappa}{4\pi}\epsilon^{\mu\nu\rho}\mathrm{Tr}\left(A_\mu\partial_\mu A_\rho - \frac{2i}{3}A_\mu A_\nu A_\rho\right) + \bar{\psi} i \not{\mathcal{D}}\psi - \mathcal{D}^\mu\bar{\phi}\mathcal{D}_\mu\phi + \frac{4\pi^2}{\kappa^2}(\bar{\phi}\phi)^3 + \frac{4\pi}{\kappa}(\bar{\phi}\phi)(\bar{\psi}\psi) + \frac{2\pi}{\kappa}(\bar{\psi}\phi)(\bar{\phi}\psi)\Big]$$

$$\kappa \to -\kappa, \lambda \to \lambda - \mathrm{sgn}(\lambda)$$

Recursion Relations for Tree-level Amplitudes

[arXiv: 1710.04227 K.Inbasekar, S.Jain, **PN**, V.Umesh]

- By deforming external momenta in complex plane, a higher point amplitude factorizes into sum over product of lower point amplitudes



$$A_{2n}(z=1) = \sum_f \int \frac{d\theta}{p_f^2}\left(z_{a;f}\frac{z_{b;f}^2 - 1}{z_{a;f}^2 - z_{b;f}^2} A_L(z_{a;f},\theta) A_R(z_{a;f}, i\theta) + (z_{a;f} \leftrightarrow z_{b;f}) \right)$$

$z_{a,b;f}$ are the zeros of $p_f^2(z) = 0$

Recursion Relations in non-SUSY theories

- Tree-level diagrams of fermion-only scattering amplitudes are same between SUSY/non-SUSY (fermion-coupled to CS) theory
- SUSY ward identity: 4-point super-amplitude is completely determined by 4-fermion scattering amplitude
  This implies, an arbitrary higher-point tree-level amplitude can be written only in terms of 4-fermion amplitude
- Hence, an arbitrary higher-point tree-level amplitude in fermion-coupled to CS theory can be written only in terms of 4-point fermion amplitude

4-pt Function in $\mathcal{N} = 2$ Theory

- Non-renormalization of 4-point amplitudes

  $2 \to 2$ scattering matrices are tree-level exact in $\mathcal{N} = 2$ theories [Inbasekar, Jain, Mazumdar, Minwalla, et al. '15],

$$T^{non-anyonic}_{all-loop} = T_{tree} = \frac{4\pi}{\kappa} \frac{\langle 12 \rangle}{\langle 23 \rangle} \delta^{(3)}(P)\, \delta^{(2)}\left(\mathcal{Q}\right)$$

  except in anyonic-channel,

$$T^{anyonic}_{all-loop} = N \frac{\sin(\pi\lambda)}{\pi\lambda} T_{tree}$$

- Such non-renormalization in known examples is a result of some symmetries

Our work is directed towards understanding such hidden symmetries!

Dual-Superconformal Symmetry

arXiv: 1711.02672 K.Inbasekar, S.Jain, S. Majumdar, **PN**, T. Neogi, T. Sharma, R. Sinha, V.Umesh

- The non-renormalization of the 4-point function in $\mathcal{N} = 2$ theory isn't well explained.
- There is reason to believe that there might be some additional symmetry in the theory!

Dual-Superconformal Symmetry

- Define dual variables as follows:

$$x^{\alpha\beta}_{i,i+1} = x^{\alpha\beta}_i - x^{\alpha\beta}_{i+1} = p^{\alpha\beta}_i,$$
$$\theta^{\alpha}_{i,i+1} = \theta^{\alpha}_i - \theta^{\alpha}_{i+1} = q^{\alpha}_i$$

- The 4-point scattering amplitude can be written in terms of the dual variables:

$$\mathcal{A}_4 = \sqrt{\frac{x^2_{1,3}}{x^2_{2,4}}}\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$$

Dual-Superconformal Symmetry

- $$\left(K^{\alpha\beta} + \frac{1}{2}\sum_{j=1}^{4} \Delta_j x_j^{\alpha\beta}\right) \mathcal{A}_4 \ = 0, \ \left(\bar{S}_\alpha + \frac{1}{2}\sum_{j=1}^{4} \Delta_j \theta_{j\alpha}\right) \mathcal{A}_4 \ = 0$$

  where, $K_{\alpha\beta}$ and $S_{\alpha\beta}$ are the generators in superconformal algebra:

  $$K^{\alpha\beta} = IP_{\alpha\beta}I \ , \ \ \bar{S}_\alpha = I\bar{Q}_\alpha I$$

  with,

  $$P_{\alpha\beta} = \sum_{j=1}^{4} \frac{\partial}{\partial x_i^{\alpha\beta}}, \ \ \bar{Q}_\alpha = \sum_{j=1}^{4} \theta_i^\beta \frac{\partial}{\partial x_i^{\beta\alpha}}$$

- With particular choice of dual coordinates we have $\Delta_i = \{3, 1, -1, 1\}$

Interesting Future Directions

- Exact computation of all-loop higher point amplitudes
  - establishing the strong-weak coupling duality for higher-point amplitudes
  - understand their non-renormalization
  - understand their symmetries
- More interesting anyonic behaviour of higher-point functions can be used to compute the Aharonov-Bohm phases in multi-particle states.