All-loop cut from the Amplituhedron

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Amplituhedron

- L-loop amplituhedron $\mathcal{A}_{n,k,L}$ is space of all k-planes Y in $(k+4)$-dimensions and L 2-planes $(AB)_i = \mathcal{L}_i$ in the 4-dimensional complement of Y such that

$$Y = C \cdot Z, \qquad \mathcal{L}_i = D_{(i)} \cdot Z \tag{1}$$

  where $C \in G_+(k,n)$, the $D_{(i)} \in C^\perp$ are $2 \times n$ matrices satisfying extended positivity conditions with C and the external data $Z \in M_+(n, k+4)$ is positive: $\langle Z_{a_1} \cdots Z_{a_{k+m}} \rangle > 0$ for $a_1 < \cdots < a_{k+m}$
- The superamplitude is extracted from a canonical form $\Omega_{n,k,L}(\mathcal{Y}, Z)$ with logarithmic singularities on the boundaries of the amplituhedron
- To determine Ω we "triangulate" or "cellulate" the space by solving inequalities; we don't know how to triangulate the space for general n, k, L

Alternate characterization of $\mathcal{A}_{n,k,L}$

- There is another characterization of the amplituhedron in terms of projection and sign flips: for each $(k+2)$-plane $(YAB)_i$ we can project through $(YAB)_i$ or simply Y and we must end up in $m=2, k+2$ and $m=4, k$ amplituhedra, respectively
- Phrasing this in terms of sign flips, we have the conditions

$$\begin{aligned} \langle (YAB)_\ell i i{+}1\rangle > 0, \quad \langle Y i i{+}j j{+}1\rangle > 0, \\ \{\langle (YAB)_\ell 12\rangle, \ldots, \langle (YAB)_\ell 1n\rangle\} \quad \text{has } k+2 \text{ sign flips}, \\ \{\langle Y1234\rangle, \ldots, \langle Y123n\rangle\} \quad \text{has } k \text{ sign flips}, \end{aligned} \tag{2}$$

  for each $\ell = 1, \ldots, L$

- We will focus on $k=0$ or parity conjugate, for this the form depends only on L lines $(AB)_\alpha$, $\alpha = 1, \ldots, L$
- After projection each $(AB)_\alpha$ must be in the 1-loop amplituhedron
- Mutual positivity condition between loops,

$$\langle (AB)_\alpha (AB)_\beta \rangle > 0, \tag{3}$$

  for all pairs α, β.

- We don't know how to triangulate the space for general n, L, so look at particular cut of the amplitude where the problem factorizes and we can get L-loop information

L-loop intersecting cut

- We consider the cut where all lines intersect in a common point, say, A, so that

$$\boxed{\langle (AB)_\alpha (AB)_\beta \rangle = 0 \quad \text{for all pairs } \alpha, \beta} \tag{4}$$

- For $k = 0$ the inequalities demand that all sequences of the form

$$\{\langle AB_\alpha 12\rangle, \ldots, \langle AB_\alpha 1n\rangle\} \tag{5}$$

  have exactly 2 sign flips

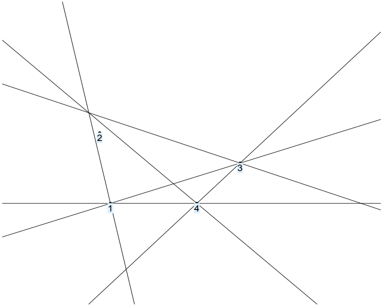
- Instead of summing over different flip options we will consider the parity conjugate ($\overline{\text{MHV}}$) problem where we simply require

$$\boxed{\langle AB_\alpha ij\rangle > 0 \qquad \text{for all } i, j} \tag{6}$$

- The key feature of the cut: the form in B_α factorizes:

$$\Omega = \sum_A \Omega_A \prod_{\alpha=1}^{L} \Omega_{B_\alpha}. \tag{7}$$

Projecting through A



- Project through A, B_α is in region satisfying $\boxed{\langle AB_\alpha ij\rangle > 0}$
- Drawing all such pictures for configurations of n-points we triangulate the space

- Naively the region triangulated by the inequalities $\langle AB_\alpha ij\rangle > 0$ could be anything from a triangle to an n-gon; however demanding consistency with positive data kills all configurations except triangles in B_α!

$\Longrightarrow$ the form is given by all triangles in B_α along with corresponding forms in A for all configurations consistent with the inequalities+positivity

Bootstrapping the form at L-loops

- Bootstrap L-loops directly from 2-loops; if we know the answer at $L = 2$ expressed as a sum of triangles, it lifts directly to any L
- For $\overline{\text{MHV}}$ the 2-loop integrand is a sum of double-boxes and (parity-conjugated) penta-boxes and double-pentagons; by taking cuts in B_α of this result we isolate the coefficients of triangles

For triangles $(i{-}1i) - (ii{+}1) - (i{+}1i{+}2)$ the geometry in A is a tetrahedron

$$\frac{\langle i{-}2i{-}1ii{+}1\rangle\langle i{-}1ii{+}1i{+}2\rangle\langle ii{+}1i{+}2i{+}3\rangle}{\langle Ai{-}2i{-}1i\rangle\langle Ai{-}1ii{+}1\rangle\langle Aii{+}1i{+}2\rangle\langle Ai{+}1i{+}2i{+}3\rangle}\prod_\alpha\frac{\langle Ai{-}1ii{+}1\rangle\langle Aii{+}1i{+}2\rangle}{\langle AB_\alpha i{-}1i\rangle\langle AB_\alpha ii{+}1\rangle\langle AB_\alpha i{+}1i{+}2\rangle}. \tag{8}$$

For triangles $(i{-}1i) - (ii{+}1) - (jj{+}1)$ the geometry in A is a cyclic polytope

$$\frac{\langle Aijj{+}1\rangle\langle i{-}2i{-}1ii{+}1\rangle\langle i{-}1ii{+}1i{+}2\rangle\langle j{-}1jj{+}1j{+}2\rangle}{\langle Ai{-}2i{-}1i\rangle\langle Ai{-}1ii{+}1\rangle\langle Aii{+}1i{+}2\rangle\langle Aj{-}1jj{+}1\rangle\langle Ajj{+}1j{+}2\rangle}\prod_\alpha\frac{\langle Ai{-}1ii{+}1\rangle\langle Aijj{+}1\rangle}{\langle AB_\alpha i{-}1i\rangle\langle AB_\alpha ii{+}1\rangle\langle AB_\alpha jj{+}1\rangle}. \tag{9}$$

For triangles $(ii{+}1) - (jj{+}1) - (kk{+}1)$ the geometry is an octahedron

$$\begin{gathered}\frac{\langle A(ii{+}1)\cap(Ajj{+}1)kk{+}1\rangle\langle i{-}1ii{+}1i{+}2\rangle\langle j{-}1jj{+}1j{+}2\rangle\langle k{-}1kk{+}1k{+}2\rangle}{\langle Ai{-}1ii{+}1\rangle\langle Aii{+}1i{+}2\rangle\langle Aj{-}1jj{+}1\rangle\langle Ajj{+}1j{+}2\rangle\langle Ak{-}1kk{+}1\rangle\langle Akk{+}1k{+}2\rangle}\\ \times\prod_\alpha\frac{\langle A(ii{+}1)\cap(Ajj{+}1)kk{+}1\rangle}{\langle AB_\alpha ii{+}1\rangle\langle AB_\alpha jj{+}1\rangle\langle AB_\alpha kk{+}1\rangle}.\end{gathered} \tag{10}$$

Form at n-points

The form at n-points is given by summing over all possible triangles,

$$\Omega = \frac{1}{2} \sum_{i<j<k} \frac{\langle A(ii{+}1) \cap (Ajj{+}1)kk{+}1\rangle \langle i{-}1ii{+}1i{+}2\rangle \langle j{-}1jj{+}1j{+}2\rangle \langle k{-}1kk{+}1k{+}2\rangle}{\langle Ai{-}1ii{+}1\rangle \langle Aii{+}1i{+}2\rangle \langle Aj{-}1jj{+}1\rangle \langle Ajj{+}1j{+}2\rangle \langle Ak{-}1kk{+}1\rangle \langle Akk{+}1k{+}2\rangle}$$
$$\times \prod_{\alpha} \frac{\langle A(ii{+}1) \cap (Ajj{+}1)kk{+}1\rangle}{\langle AB_\alpha ii{+}1\rangle \langle AB_\alpha jj{+}1\rangle \langle AB_\alpha kk{+}1\rangle}. \tag{11}$$

- We checked this through $n = 9$ points against the local expansion at two-loops, and are working on n-point proof by comparing on cuts
- Outlook: how to lift this cut to $\langle (AB)_\alpha (AB)_\beta \rangle \neq 0$? Is there a form which naturally descends to this sum-of-triangles on the intersecting cut?

Thank you for your 4 minutes!