Scattering on plane waves from ambitwistor strings

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Plane waves are highly symmetric spacetimes

This allows us to define an analogue of flat space momentum eigenstates for spin 0, 1 and 2 fields

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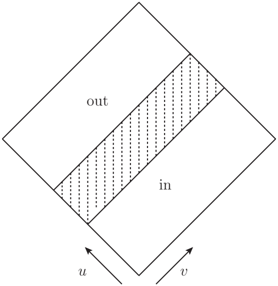
Sandwich plane wave

$$ds^2 = 2\,du\,dv - H_{ab}(u)\,x^a\,x^b\,du^2 - dx_a\,dx^a$$

$$R^a{}_{ubu} = -H^a_b(u)$$



Symmetries and momentum eigenstates

$$ds^2 = 2\,dU\,dV - \gamma_{ij}(U)\,dy^i\,dy^j$$

The 2d-3 Killing vectors form a Heisenberg algebra, the obvious ones are

$$\partial_V\,, \quad \partial_i$$

The wave equation $\Box\Phi = 0$ is solved by

$$\Phi(X) = \Omega(U)\,e^{i\,\phi_k}\,,$$

$$\phi_k = k_0\,v + k_i\,y^i + \frac{k_i k_j F^{ij}(U)}{2\,k_0}$$

Spin 1 and 2 solutions can be generated by acting with a spin raising operator

Scattering is well defined on plane wave spacetimes

Using the in and out states we show that time evolution is unitary and there is no particle creation

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In the flat in and out regions, we have the standard QFT notion of in and out states

An analouge of the Klein-Gordon inner product can be defined using the foliation by hypersurfaces Σ_u of constant u:

$$\langle \Phi_1 | \Phi_2 \rangle = i \int_{\Sigma_u} \mathrm{d}v \, \mathrm{d}^{d-2}x \, \left(\Phi_1 \, \partial_v \bar{\Phi}_2 - \bar{\Phi}_2 \, \partial_v \Phi_1 \right)$$

Positive frequency states satisfy

$$\left\langle \Phi_1^{\mathrm{in}} | \Phi_2^{\mathrm{in}} \right\rangle = 2 \, k_0 \, \delta(k_0 - l_0) \, \delta^{d-2}(k_i - l_i)$$

$$\left\langle \Phi_1^{\mathrm{out}} | \bar{\Phi}_2^{\mathrm{in}} \right\rangle = 0.$$

Therefore the evolution is unitary and there is no particle creation as already found in [Gibbons 75]. This argument generalises to spin 1 and 2.

Ambitwistor strings provide the correct 3-point amplitudes

The calculations are much simpler than expected as all quantum corrections drop out

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Ambitwistor strings are worldsheet theories that can be used to obtain QFT amplitudes

We calculated 3-point graviton amplitudes on a plane wave background using the curved space ambitwistor string [Adamo et al. 15]. The three point correlation function of vertex operators yields

$$\langle V_1(z_1)\, V_2(z_2)\, c(z_3)\tilde{c}(z_3)\, U_3(z_3)\rangle = \delta^{d-1}\left(\sum_{r=1}^{3} k_r\right) \int \frac{\mathrm{d}u}{\sqrt[4]{|\gamma|}} \Big[(\varepsilon_1 \cdot \varepsilon_3\, K_1 \cdot \varepsilon_2 + \text{cyclic})^2 - i\, k_{1\,0} k_{2\,0} k_{3\,0}\, \sigma^{ab} \mathcal{C}_a \mathcal{C}_b\Big] \exp\left(i F^{ij} \sum_{s=1}^{3} \frac{k_{s\,i} k_{s\,j}}{2 k_{s\,0}}\right),$$

- This agrees with the curved spacetime QFT calculation
- BRST closure of the vertex operators fixes the external states

Similar calculations can be done for a gauge field on a plane wave gauge field background

These 3-point amplitudes obey a double copy relation

Gravity=YM^2 holds if one squares the background and the dynamical fields in the background

Is there a notion of double copy?

- Naïve attempt: Double copy gauge amplitude on same plane wave spacetime
- Fails!

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Not surprising. We know

- Flat space amplitudes: gravity=ym^2
- Various examples where backgrounds satisfy Gravity=YM^2
- Therefore expect:

$$(\text{Gravity} + \text{gravity}) = (\text{YM} + \text{ym})^2$$

- This works!
- Subtlety: relating objects on flat space to objects on curved spacetime
- Careful prescription required, e.g. external momenta

Further questions

- Does the double copy relation hold for higher point amplitudes
- Higher point amplitudes from ambitwistor strings
- Different backgrounds
- ...

Thank you!